ENGR I1100: Fall 2025
Introduction to Engineering Analysis
<u>Computer Project – report due December 8, 2025</u>

This project addresses the heat equation as well as the wave equation, both in 1D on a finite domain. The recommended computational environment is MATLAB where the calculations and visualization are easily combined. However, you may perform the computation and plotting in another environment (e.g. FORTRAN or C, or Python; Mathematica possible).

Consider the following sequence of problems, writing a short report on your work and your findings based on the results in each portion. State your tools used (software, in particular) within the report (not as a reference) and properly reference at end of report any sources of information used.

1. **1D Heat equation with constant coefficients.**

For a field $u(x,t)$, set up a discretization of the domain $-L < x < L$. You will want to be able to vary the number of points in the x-domain to establish the role of spatial discretization, $\Delta x = 2L/N$, where N is to be varied (this assumes constant Δx, which is probably best for this project). Time will also be discretized, so you will need to experiment with the values of Δt.

Consider the problem:

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$$
$$u(x,t=0) = H[1-(x/L)^2]$$
$$u(x=L) = u(x=-L) = 0$$

Evaluate the approach, based on a numerical discretization, to the steady state solution. Compare this with the analytical solution using varying numbers of terms in the series solution. As noted, you should vary N.

2. **Nonlinear heat transfer, *k*(T)**

Allow the coefficient k to depend on temperature. Here, consider
A) the experimentally determined value for a real material (a metal or metal oxide, for example) over a wide range of temperatures and
B) an extreme variation, e.g. varying as T^{a}

3. Wave equation

Consider the problem

$$\frac{\partial^2 A}{\partial t^2} = c^2 \frac{\partial^2 A}{\partial x^2},\ A(0) = 0,\ A(1) = 0;\ A(t=0) = 2x(1-x),\ dA/dt(x=0) = 0.$$

a) Compute the solution as done for the heat equation in 1, for constant wave speed $c = 1$, and compare to the analytical solution.
b) Let the wave speed c be a function of A: compute and specifically report on the case $c = 1 + \alpha A$, allowing α to first be small (e.g. $\alpha = 0.01$), and then increasing it (e.g., $\alpha = 0.25, 1$) and determining the behavior. Be alert to values of α for which the solution seems to become unphysical. You are, of course, free to try other forms of c for your own education and entertainment.

REPORTING: Prepare a short report addressing the comparison of the numerical and analytical solutions for case 1, and the salient features deduced from your examinations in 2. Restrict this to 6 figures (6 plots, not multipart figures) and 4 pages of text single spaced; references (which should be included for any sources of information used) do not count against this page count. This is to be submitted electronically.