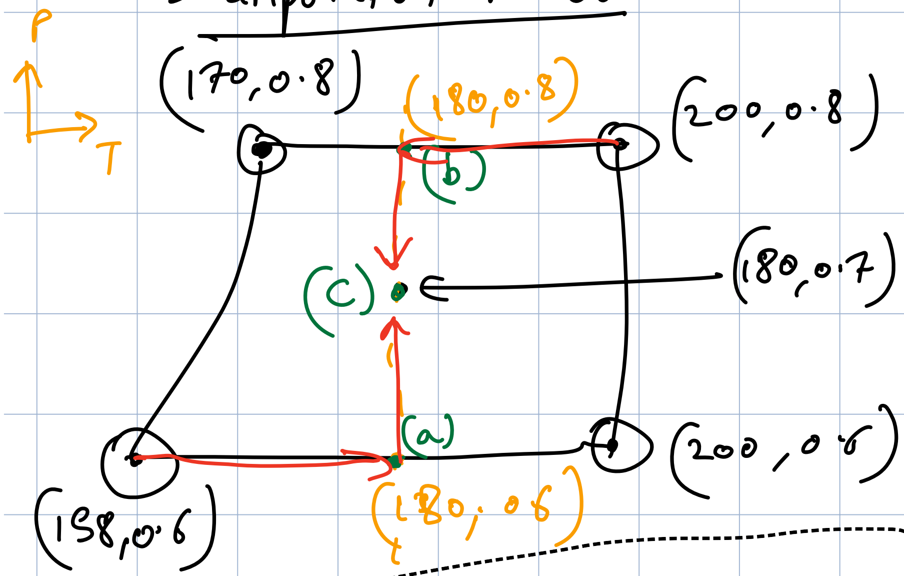
Thermo

Recitation

#2

- → interpolation review
- → Problems : 2.2, 2.16, 2.21, 2.18

Interpolation review



	(x_1)	(x_2)	x	y_1	y_2	y
P(MPa)	T_1	T_2	T_3	u_1	u_2	u_3
x_1 0.6	158.8	200	180	2566.8	2639.3	(a) y_1 ✓
0.7	180	180	180	×	×	(c)
x_2 0.8	170.4	200	180	2576	2631	(b) y_2 ✓

@ 0.6 MPa

$u_2 - u_1 / T_2 - T_1$

(a) $m = y_2 - y_1 / x_2 - x_1$

$y = m(x - x_2) + y_2$

x & y in red

$\Rightarrow u_3 = m(T_3 - T_2) + u_2$

u_3 @ 180° & 0.6 MPa = 2604.1 kJ/kg

(a)

@ 0.8 MPa

(b) Same formula.

u_3 @ 180° & 0.8 MPa = 2593.83 kJ/kg

(b)

@ 0.7 MPa

(c) x & y in pink

Same formula. ← remember.

u_3 @ 180° & 0.7 MPa = 2598.95 kJ/kg

(c) ← desired value.

2.2

Given:

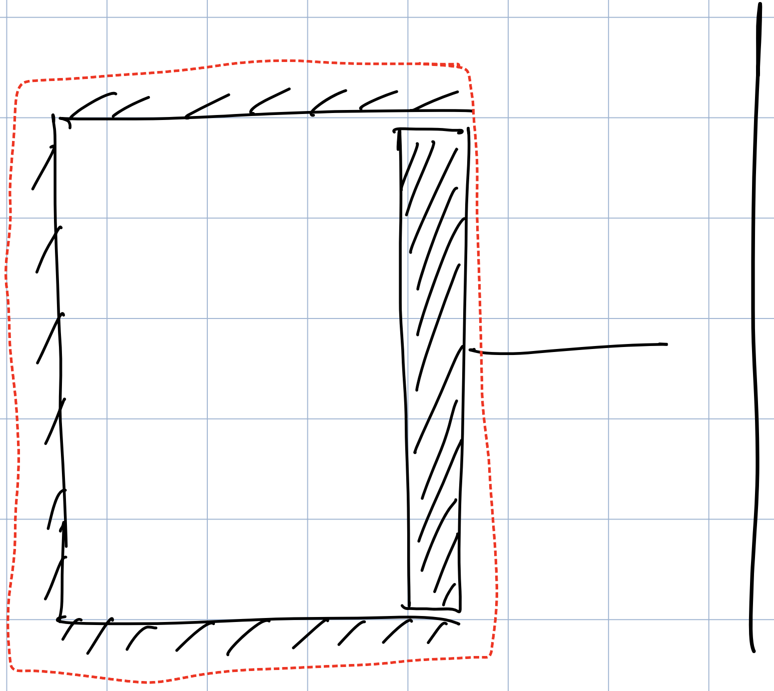
$n = 1 \text{ mol}$

$C_p = \frac{7R}{2}$ (diatomic gas)

$T_i = 100\text{ K}$

$P_i = 0.1\text{ MPa}$

$P_f = 0.7\text{ MPa}$



Find:

ΔU, ΔH, Q, W_{EC}

For monoatomic

$C_p = \frac{5R}{2}$

$\Rightarrow$ Assume : ideal gas (diatomic)
closed system
Rigid boundaries.

(a) isothermal

i.e $\Delta T = 0$

$\therefore T_i = T_f = 100\,K$

$P_i = 0.1\,MPa$ & $P_f = 0.7\,MPa$

For an ideal, isothermal, closed system: $\Delta U = 0$, $\Delta H = 0$

$\Delta U = Q + W_{EC}$ $\leftarrow$ work done on the gas

$\because \Delta U = 0$

$\therefore Q = -W_{EC}$

work on the system is given by $\Rightarrow \underline{W}_{EC} = -\int P\,d\underline{V}$

$$\because P\underline{V} = nRT$$

$$\therefore P = \frac{nRT}{\underline{V}}$$

$$\therefore W_{EC} = -\int_{\underline{V_i}}^{V_f} \frac{nRT}{\underline{V}}\,d\underline{V} = -nRT\int_{\underline{V_i}}^{V_f} \frac{d\underline{V}}{\underline{V}} = -nRT\left[\ln \underline{V_f} - \ln \underline{V_i}\right]$$

$$= -nRT\ln\left(\frac{V_f}{V_i}\right) = -nRT\ln\left(\frac{P_i}{P_f}\right)$$

$$\underline{V_1} = \frac{nRT}{P_1}, \quad \underline{V_2} = \frac{nRT}{P_2}$$

$$nRT = \underline{V_1}P_1 = \underline{V_2}P_2$$

$$\boxed{\frac{V_2}{V_1} = \frac{P_1}{P_2}}$$

This correlation is valid at const. temp

$$W_{EC} = -nRT\ln\left(\frac{0.1}{0.7}\right)$$

$$= -\underset{\cancel{mol}}{1} \times \underset{\frac{J}{\cancel{mol}\cdot\cancel{K}}}{8.314} \times \underset{\cancel{K}}{100} \times \ln\left(\frac{0.1}{0.7}\right)$$

$$\underline{W}_{EC} = 1618\ J$$

$$\therefore \underline{Q} = -1618\ J$$

b) isochoric $\Rightarrow \Delta V = 0 \Rightarrow v_i = v_f$

$$W_{EC} = -P\Delta V$$

$$\therefore W_{EC} = -\int P\,\cancel{dV}^{\,0} = 0$$

$$\therefore \Delta U = Q + \cancel{W_{EC}}^{\,0}$$

$$\therefore \Delta U = Q = nC_V \underset{(\underline{T_f} - T_i)}{\underset{\uparrow}{\Delta T}}$$

For T_f :

$$v_i = v_f = \frac{\cancel{n}\cancel{R}T_i}{P_i} = \frac{\cancel{n}\cancel{R}T_f}{P_f}$$

$$\therefore T_f = \frac{T_i P_f}{P_i} = \frac{100 \times 0.7}{0.1} = 700\,K$$

For <u>Diatomic gas</u>

$$C_p = \frac{7R}{2}$$

$$\because C_p = C_v + R$$

$$\therefore C_v = \frac{7R}{2} - R$$

$$\therefore C_v = \frac{5R}{2}$$

$$\therefore \quad \Delta U = n C_v (700 - 100)$$

$$= 1 \times \frac{5R}{2} \times 600$$

$$\cancel{\text{mol}} \quad \frac{\text{J}}{\cancel{\text{mol}}\ \cancel{\text{K}}} \quad \cancel{\text{K}}$$

$$\Delta U = 12471 \text{ J}$$

$$\therefore \underline{\Delta U} = \underline{q} = 12{,}471 \text{ J}$$

$$\underline{\Delta H} = n C_p \Delta T$$

$$= 1 \times \frac{7R}{2} \times (T_f - T_i)$$

$$= 1 \times \frac{7R}{2} \times 600$$

$$\therefore \underline{\Delta H} = 17{,}459 \text{ J}$$

c) Adiabatic $\Rightarrow$ $Q = 0\,J$

equation:

$$\because \quad \Delta U = \overset{0}{\cancel{Q}} + W_{EC} \rightarrow P\Delta V$$

$$\Delta U = n\,C_v\,\Delta T$$

$$\Delta H = n\,C_p\,\Delta T$$

Now, $\Delta U = W_{EC}$

$$n\,C_v\,\Delta T = -P\,\Delta V$$

Taking limit & integrating on both sides

$$\therefore \int n\,C_v\,dT = -\int P\,dV \qquad P = \frac{nRT}{V}$$

$$\therefore n C_v \int dT = -nRT \int \frac{dV}{V}$$

$$\therefore \int dT = \frac{-nRT}{nC_v} \int \frac{dV}{V}$$

$$\int \frac{dT}{T} = \frac{-R}{C_v} \int \frac{dV}{V}$$

$$\ln \frac{T_2}{T_1} = \frac{-R}{C_v} \ln \frac{V_2}{V_1}$$

$$\ln \frac{T_2}{T_1} = \frac{R}{C_v} \ln \frac{V_1}{V_2}$$

$$\therefore \frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{R/C_v}$$

For adiabatic process

$$PV^{\gamma} = \text{Const}, \quad \gamma = C_p/C_v$$

$$\therefore P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$$

$$\therefore \frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{R/C_V}$$

$$\therefore \frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{R}{\cancel{C_V}} \times \frac{\cancel{C_V}}{C_P}}$$

$$\therefore T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{R/c_p}$$

$$= 100 \left(\frac{0.7}{0.1}\right)^{R/\frac{7}{2}R}$$

$$\therefore T_2 = 100\,(7)^{2/7} = 174.4\,K$$

$$\therefore \Delta T = T_2 - T_1 = 74.4\,K.$$

$$\therefore \frac{P_1}{P_2} = \left(\frac{V_2}{V_1}\right)^{\gamma}$$

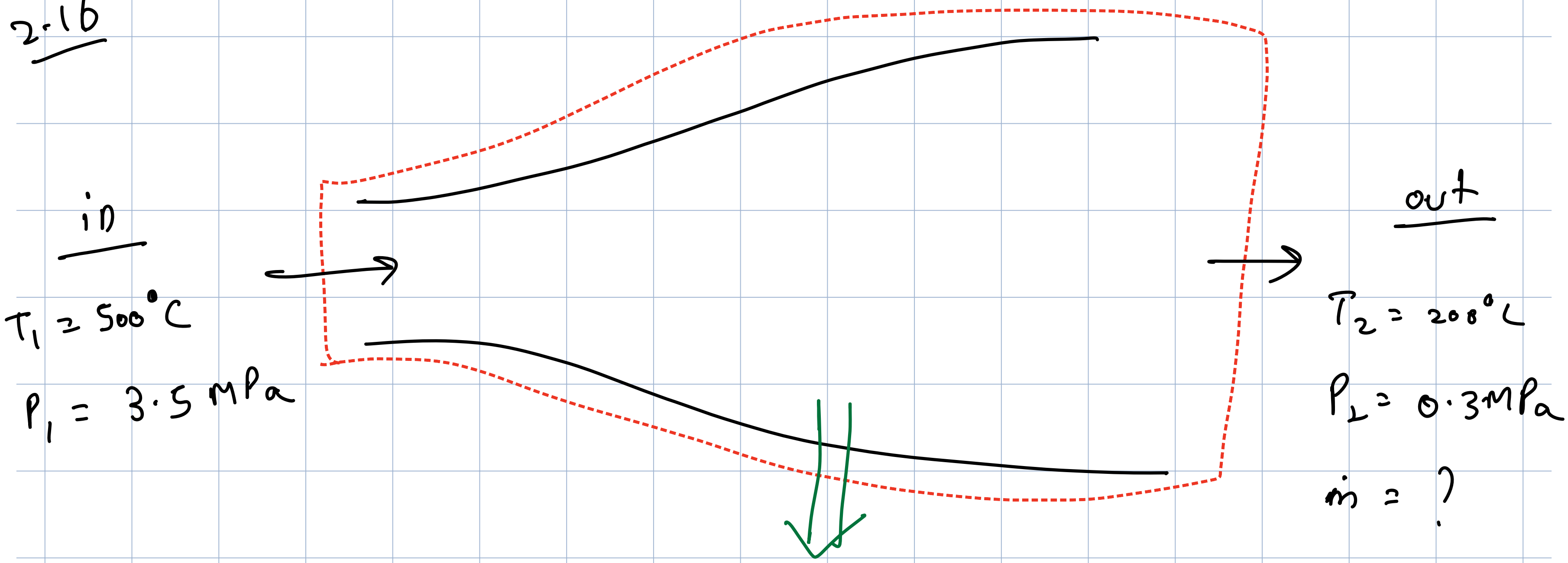
$$\therefore \frac{V_1}{V_2} = \left(\frac{P_2}{P_1}\right)^{1/\gamma}$$

$$\boxed{\frac{V_1}{V_2} = \left(\frac{P_2}{P_1}\right)^{c_v/c_p}}$$

$\therefore \quad \underline{\Delta U} = n C_v \Delta T = 1 \times \frac{5}{2} \times R \times 74.4 = 1546.40\,J$

$\therefore \quad \underline{\Delta H} = n C_p \Delta T = 1 \times \frac{7}{2} R \times 74.4 = 2164.96\,J$

2.16



$T_1 = 500°C$

$P_1 = 3.5\,MPa$

$T_2 = 208°C$

$P_2 = 0.3\,MPa$

$\dot{m} = ?$

$|\dot{W}_s| = 750\,kW$

$\dot{W}_s = -750\,kJ/s$

$\Rightarrow$ Adiabatic $\rightarrow$ $\dot{Q} = 0$

steady state $\Rightarrow$ Accumulation = 0

open system $\Rightarrow$ $\dot{W}_{Ec}$

Mass balance

$$\dot{m}_{in} = \dot{m}_{out} = \dot{m}$$

Energy balance

$$0 = H^{in}\dot{m}^{in} - H^{out}\dot{m}^{out} + \dot{W}_s + \cancelto{0}{Q} + \cancelto{0}{W}_{EC}$$

$$0 = \left(H^{in} - H^{out}\right)\dot{m} + \dot{W}_s$$

$$0 = -\Delta H\,\dot{m} + \underline{\dot{W}_s}$$

$$\therefore \quad \dot{m} = \frac{\dot{W}_s}{\Delta H}$$

	$T_1 = 500\,°C$ $P_1 = 3.5$ MPa (in)	$T_2 = 200\,°C$ $P_2 = 0.3$ MPa (out)
H (kJ/kg)	3451.6	2865.9

$$\dot{m} = \frac{-750 \; \cancel{kJ}/s}{(2865.9 - 3451.6) \; \cancel{kJ}/kg} = 1.26 \; kg/s$$

2.21

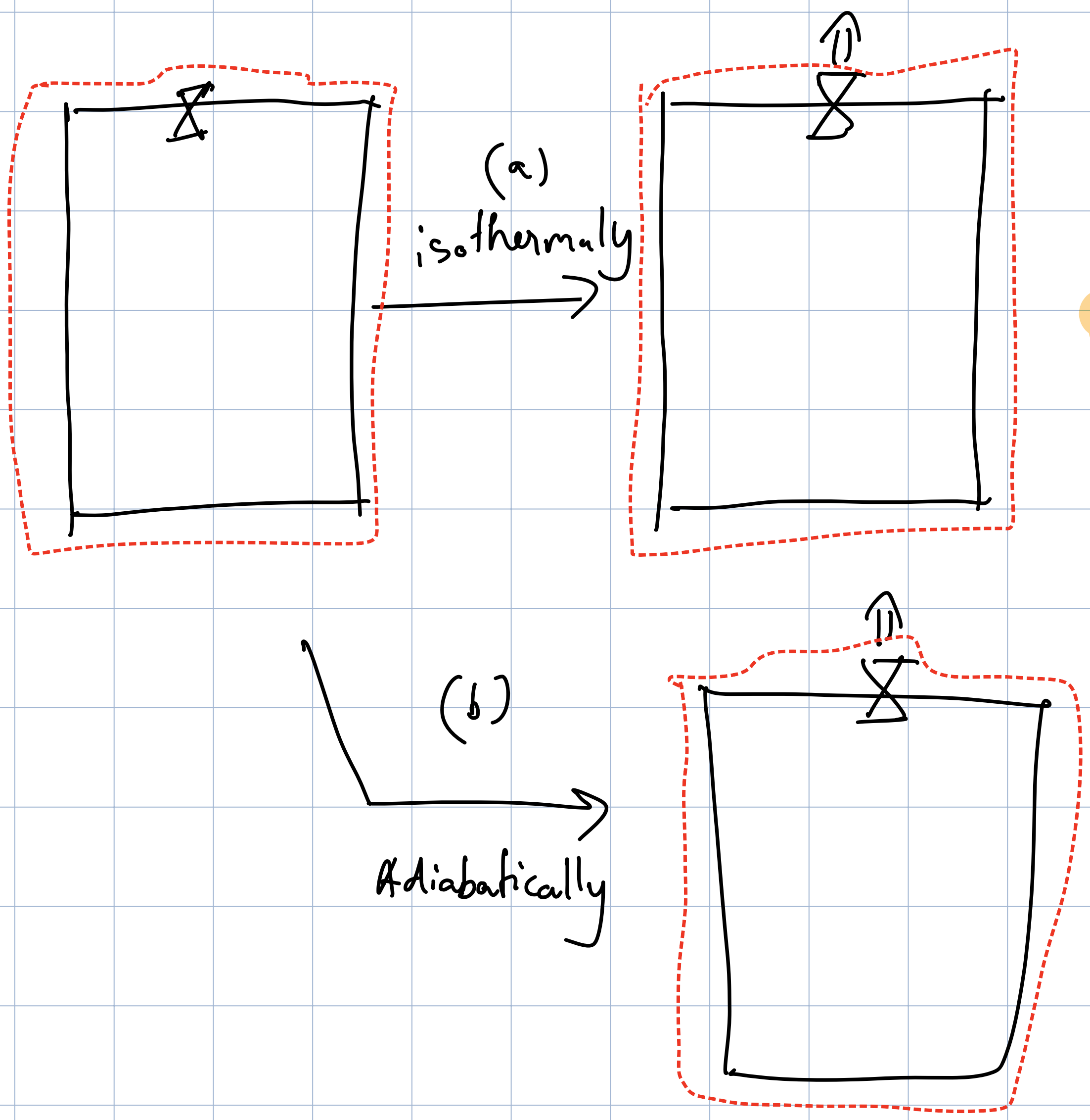
initially

$V_1 = 0.1 m^3$

$P_1 = 10$ bar

$= 1 MPa$

$T_1 = 300 K.$



Final

$P_2 = 1 bar = 0.1 MPa$

$T_2 = 300 k$

$n_2 = ?$

$P_2 = 1 bar = 0.1 MPa$

$T_2 = ?$

$n_2 = ?$

$\Rightarrow$ Assume ideal gas.

$\because$ $C_p/R = 3.5$ $\Rightarrow$ diatomic gas.

Finding n_1 :

$$n_1 = \frac{P_1 V}{R T_1} = \frac{\overset{\cancel{Pa}}{1\times10^6} \times \overset{\cancel{m}3}{0.1}}{\underset{\frac{\cancel{m^3}\,\cancel{Pa}}{mol\,\cancel{K}}}{8.314} \times \underset{\cancel{K}}{300}}$$

$$\therefore n_1 = 40 \text{ mols}$$

(a) isothermal $\Rightarrow$ $\Delta T = 0$

$$n_2 = \frac{V P_2}{R T_2} = \frac{0.1 \times 10^5}{8.314 \times 300} \approx 4.01 \text{ mols}$$

(b) Adiabatic $\Rightarrow$ $Q = 0$

we know, $PV^{\gamma} = \text{const.}$

$$P\left(\frac{RT}{P}\right)^{\gamma} = \text{const.}$$

$$\frac{T^{\gamma}}{P^{\gamma-1}} = \text{const.}$$

$$\therefore \quad P^{1-\gamma}\cdot T^{\gamma} = \text{const.}$$

$$\therefore \quad P_1^{1-\gamma}\cdot T_1^{\gamma} = P_2^{1-\gamma}\cdot T_2^{\gamma}$$

$$\therefore \quad \left(\frac{P_1}{P_2}\right)^{1-\gamma} = \left(\frac{T_2}{T_1}\right)^{\gamma}$$

$$\frac{T_2}{T_1} = \left(\frac{P_1}{P_2}\right)^{\frac{1-\gamma}{\gamma}}$$

$$= \left(\frac{P_1}{P_2}\right)^{\left.\frac{1-C_p/C_v}{C_p/C_v}\right\}} \rightarrow \frac{C_v - C_p}{C_p}$$

$\because C_p = C_v + R$

$$= \left(\frac{P_1}{P_2}\right)^{-R/C_p}$$

$\therefore \underline{\frac{-R}{C_p}}$

$$\therefore \frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{R/C_p}$$

$$\therefore T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{R/C_p}$$

$$= 300 \left(\frac{10^5}{10^6}\right)^{R/\frac{7}{2}R}$$

$$= 300 \times (0.1)^{2/7}$$

$$\therefore T_2 = 155.4\ K$$

Now,

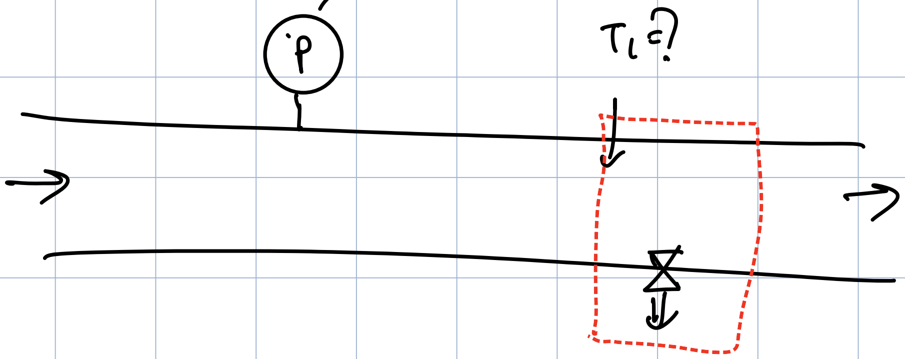
$$n_2 = \frac{V P_2}{R T_2} = \frac{0.1 \times 10^5}{8.314 \times 155.4} = 7.74 \text{ mols}$$

Extra

2.18 a)

$p = 80\,\text{bar} = 8\,\text{MPa}$

$T_1 = ?$



$T_2 = 400°C$

$P_2 = 1\,\text{bar} = 0.1\,\text{MPa}$

Assume:

Steady state.

Adiabatic $\Rightarrow Q = 0$

open system.

mass balance

$\dot{m}_{in} = \dot{m}_{out} = \dot{m}$

Energy balance

$$0 = -\Delta H_{in} \cdot \dot{m} + \cancelto{0}{\dot{W}_s} + \cancelto{0}{\dot{Q}}$$

$$\therefore \Delta H = 0$$

$$\therefore H_1 = \underset{\substack{\downarrow \\ @\ 0.1\ \text{MPa} \\ \&\ 400^\circ\text{C}}}{H_2} = 3278.6\ \text{kJ/kg}$$

Now, we need to find temperature values @ 8MPa which are closest to the H value of 3278.6 kJ/kg.

For T_1, @ 8 MPa

	T y	H x	
y_1	450	3273.3	x_1
need to find → y	?	3278.6	x (desired)
y_2	500	3399.5	x_2

interpolation

$$y = y_2 - m\,(x - x_2)$$

$$y = y_2 - \frac{y_2 - y_1}{x_2 - x_1}\,(x - x_2)$$

$$= 500 - \frac{500 - 450}{3399.5 - 3273.3}\,(3278.6 - 3399.5)$$

$$y = 547.9\ ^\circ C$$

$\therefore$ T_1 @ 8 MPa $= 547.9\ ^\circ C$

Addendum (key points to remember)

you may add these points to your cheatsheet

1 →

$$\left.\begin{aligned} \Delta U &= C_v \, \Delta T \\ \Delta H &= C_p \, \Delta T \end{aligned}\right\}$$ exact solution for ideal gas

→ For real gases only true if V, or P are constant, respectively

2 → For monoatomic gas

$$C_p = \frac{5R}{2}$$ ... (ideal gas)

For diatomic gas: $C_p = \frac{7R}{2}$ ... (ideal gas)

& $C_p - C_v = R$ ... (ideal gas)

3. → For Adiabatic process

$$\Rightarrow \quad PV^{\gamma} = \text{const.}, \quad \gamma = C_p/C_v$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{R/C_v} = \left(\frac{P_2}{P_1}\right)^{R/C_p}$$

(for ideal gases)

Derivation for $PV^{\gamma} = \text{const}$:

For ideal gas:

$$PV = RT$$

we know, $dU = dQ - dW$

for adiabatic process: $dQ = 0$

$$\therefore \quad dU = dW$$

$$\therefore \quad C_v\, dT = -P\, dV$$

substituting, $R\,\overbrace{dT} = VdP + PdV$

$$\therefore \quad V\,dP = \frac{-(R + C_V)}{C_V}\,PdV$$

$\because$ $C_p - C_V = R$ & $\gamma = C_p / C_v$ the coefficient on LHS becomes γ. Integrating on both sides

$$\therefore \quad \ln(P\underset{\nwarrow\ \text{const.}}{C}) = -\gamma \ln(V)$$

$$\therefore \quad PV^{\gamma} = \text{Const.}$$

4. Internal energy is only a function of temperature.
For ideal gas, there are no interparticle interactions
(Therefore U is not affected by pressure or volume)

$\therefore$ for isothermal process, $\Delta U = 0$
ideal, closed system.

5. Now, we know $\Rightarrow$ $H = U + PV$

$$\therefore \Delta H = \Delta U + \Delta(PV)$$

For reversible expansion/compression of an ideal gas under isothermal conditions $\Rightarrow$ $PV = \text{const.}$

$$\therefore \Delta(PV) = 0$$

& $\Delta U = 0$ (from above).

$$\therefore \Delta H = 0$$

↖ for ideal gas, isothermal, closed system