Thermo

Recitation # 4 (02/28/23)

→ Problem 2.14, 2.20 & 3 more problems

Q1

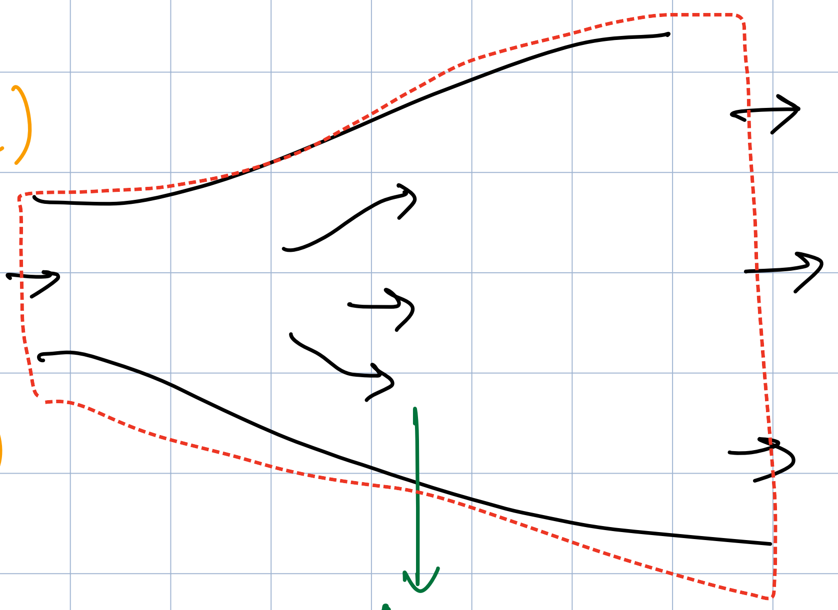
(monoatomic)

Ar

(* All noble gases are monoatomic)

$T_1 = 400\,K$

$P_1 = 50$ bar



$\dot{W}_s = ?$

$T_2 = ?$

$P_2 = 1$ bar.

Assumption → steady state

ideal gas (monoatomic → $c_p = 5R/2$)

open system.

Mass balance

$$\dot{m}_1 = -\dot{m}_2 = \dot{m}$$

Energy balance

$$0 = \dot{m} \sum_i H_i + \cancelto{0}{\underline{Q}} + \cancelto{0}{\underline{W}_{ec}} + \underline{\dot{W}_s}$$

$$0 = \dot{m}_1 H_1 - \dot{m}_2 H_2 + \underline{\dot{W}_s}$$

$$\therefore \dot{m}(H_2 - H_1) = \underline{\dot{W}_s} = \dot{m}\,\Delta H = \dot{m}\, C_p\, \Delta T$$

$$\therefore W_s = C_p \underbrace{\Delta T}_{(T_2 - T_1)}$$

↑ need to find.

we know for ideal gas 4 adiabatic process :

Remember! =)

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{R/c_V} = \left(\frac{P_2}{P_1}\right)^{R/c_p}$$

∴ we can find T_2 :

$$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{R/c_p}$$

$$\therefore T_2 = 400 \left(\frac{1}{50}\right)^{2/5}$$

$$\therefore T_2 = 83.65 \text{ K}$$

$$\therefore \Delta T = 83.65 - 400 \text{ K} = -316.35 \text{ K}$$

$$\therefore \quad \dot{W}_s = \Delta H = C_p \Delta T = \frac{5}{2} R \, (-316.35\,K)$$

$$\frac{J}{mol\,\cancel{K}} \cdot \cancel{K}$$

$$\dot{W}_s = -6575.3 \ J/mol$$

[✱ quick check :

The shaft work for system where work is derived must be -ve]

Q2

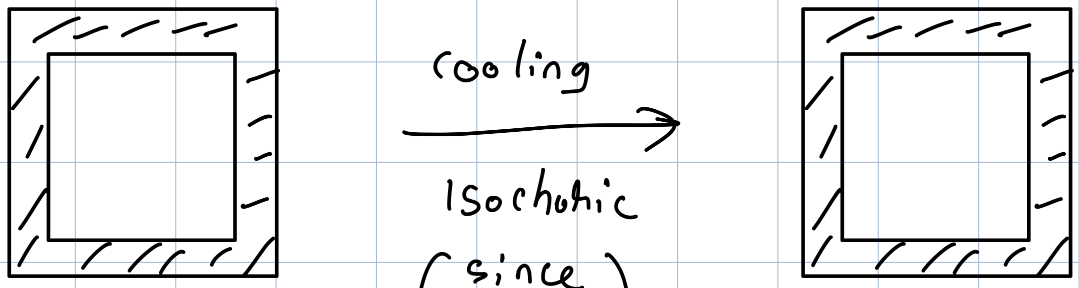
$m = 5\,kg$

$T_1 = 200°C$

$P_1 = 400\,kPa = 0.4\,MPa$.

∴ superheated.

$\therefore v_1 = 0.5343\ m^3/kg$



Isochoric

(since rigid tank)

$m = 5\,kg$

$T_2 = 60°C$

$\therefore v_1 = 0.5343\ m^3/kg$

(since isochoric)

Assumptions :

Rigid tank → $\Delta V = 0$

closed system → $\Delta m = 0$

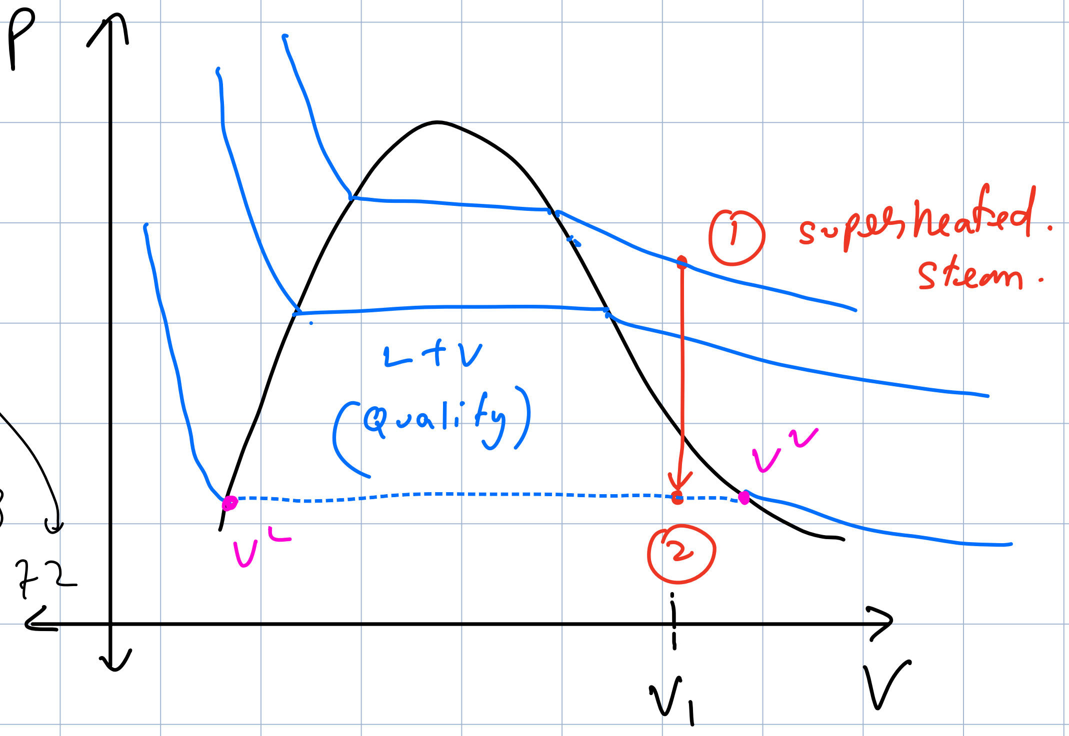
$w_s = 0$

(i) **PV diagram**

@ $T_{sat} = 60°C$

$v^L < v_1 < v^V$

$0.001017 < 0.5343 < 7.6672$



✯ we look for specific volumes at $T^{sat} = 60°C$ just to know the position of point ②. we <u>do not</u> imply that the system is at saturated condition.

(ii) **mass balance**

$m_1 = m_2 = m = 5$ kg (throughout)

Energy balance

$$\Delta U = Q + \cancelto{0}{W_{EC}} + \cancelto{0}{W_s}$$

$$\underline{\underline{\Delta U = Q}}$$

① $U_1 = 2647.2$ kJ/kg

$V_1 = 0.5343$ kJ/kg $= V_2$

② $U_2 = ?$

we find properties @ T^{sat}

$V^L = 0.001017$ m^3/kg

$V^V = 7.6672$ m^3/kg

$U^L = 251.16$ kJ/kg

$\Delta U^{vap} = 2204.74$ kJ/kg

$U^V = 2455.90$ kJ/kg

Now at point ② the quality is !

$$q = \frac{V - V^L}{V^V - V^L} = \frac{0.5343 - 0.001017}{7.6672 - 0.001017}$$

$$= 0.070$$

$$q = 7\%$$

Now, $U_2 = U^L + q \, \Delta U^{vap}$

$$= 251.16 + 0.07\,(2204.74)$$

$$U_2 = 404.5 \text{ kJ/kg}$$

$$\therefore \Delta U = U_2 - U_1 = 404.5 - 2647.2 = -2242.67 \text{ kJ/kg.}$$

$$\underline{\Delta U} = \underline{Q} = \dot{m}\,\Delta U = \underset{\cancel{\text{kg}}}{5} \times \underset{\frac{\text{kJ}}{\cancel{\text{kg}}}}{(-2242.67)}$$

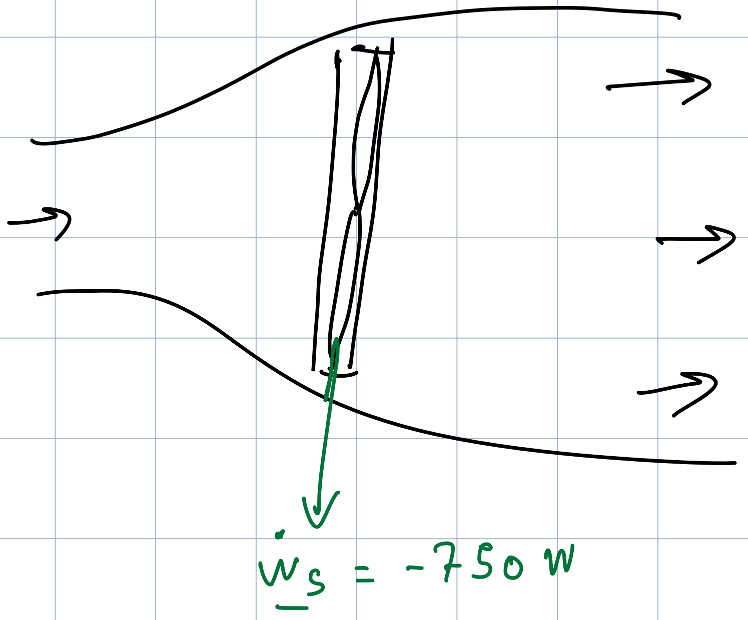
$$\therefore \underline{Q} = -11213 \text{ kJ}$$

$$\therefore \underline{Q} = -11.2 \text{ MJ}$$

[$-$Ve → implies heat is flowing out of the system.]

Q3

(a)



$T_1 = 500°C$

$P_1 = 3{\cdot}5\ MPa$

(superheated)

$T_2 = 200°C$

$P_2 = 0{\cdot}3\ MPa$

$\dot{m} = ?$

(superheated)

Assumption : Steady-state

adiabatic $\dot{Q} = 0$

Open system $\Rightarrow$ $w_{Ec} = 0$

Mass balance

$$\dot{m}_1 = -\dot{m}_2 = \dot{m}$$

Energy balance

$$0 = \dot{m}H_1 - \dot{m}H_2 + \dot{Q} + \dot{W}_s$$

$$\therefore \dot{m}\,\Delta H = \underline{\dot{W}_s}$$

$$\therefore \dot{m} = \frac{\underline{\dot{W}_s}}{\Delta H}$$

Now, $H_1 = 3451.6$ kJ/kg

$H_2 = 2865.9$ kJ/kg

$$\dot{m} = \frac{-750}{(2865.9 - 3451.6)} \; \frac{\cancel{\text{kJ}}/\text{s}}{\cancel{\text{kJ}}/\text{kg}} = 1.28 \; \frac{\text{kg}}{\text{s}}$$

$\therefore \quad \dot{m} = 1.28 \text{ kg/s}$

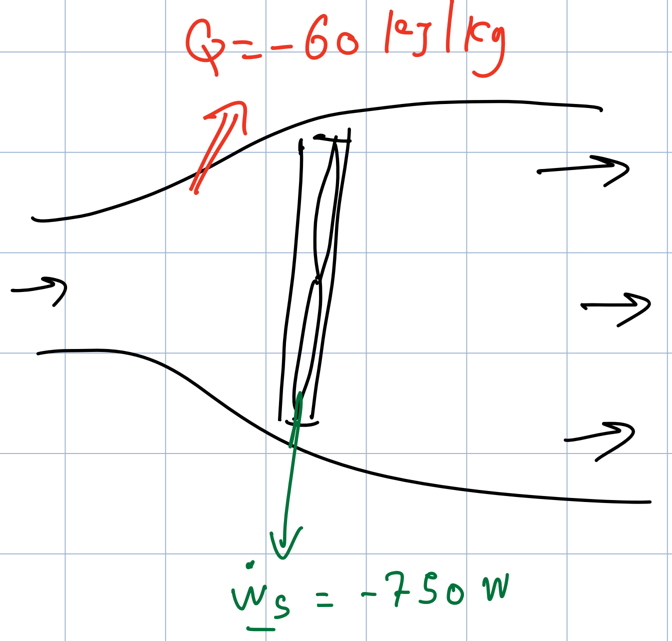
(b)

* since $Q \neq 0$ therefore adiabatic process assumption is not valid.

$T_1 = 500°C$

$P_1 = 3.5 \text{ MPa}$

(superheated)



$T_2 = \overset{150°C}{\cancel{200°C}}$

$P_2 = 0.3 \text{ MPa}$

$\dot{m} = ?$

(superheated)

mass balance

$\dot{m}_1 = -\dot{m}_2 = \dot{m}$

Energy balance

$0 = \dot{m} H_1 - \dot{m} H_2 + \underline{\dot{Q}} + \underline{\dot{W}_s}$

$\therefore \quad \underline{\dot{W}_s} = \dot{m}\,\Delta H - \underline{\dot{Q}}$

$$\therefore \quad \underline{\dot{W}_s} = \dot{m}\,(\Delta H - Q)$$

$$\therefore \quad \underline{\dot{W}_s} = 1.28\,[(2761.2 - 3451.6) - (-60)]$$

$$\therefore \quad \underline{\dot{W}_s} = -806.9 \text{ W}$$

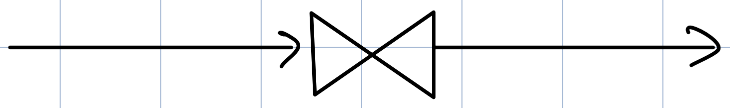
4

① $T_1 = 200°C$

$P_1 = 550$ kPa

$\dot{m} = 15$ kg/min

(superheated)



$T_2 = ?$ ②

$P_2 = 200$ kPa

(superheated)

$\Delta V = ?$

Assumption: steady-state

throttle → $\Delta H = 0$

(i)

	P(MPa)	H (kJ/kg)		U (kJ/kg)	
x_1	0.5	2855.8	y_1	2643.3	y_1
x	0.55	2853.2	y	2641.3	y
x_2	0.6	2850.6	y_2	2639.2	y_2

for H:

$$y = \left(\frac{2850.6 - 2855.8}{0.6 - 0.5}\right) \times (0.55 - 0.6) + 2850.6$$

$y = 2853.2$ kJ/kg

H @ 0.55 MPa

interpolation:

$$y = \frac{y_2 - y_1}{x_2 - x_1}(x - x_2) + y_2$$

for u :

$$y = \left(\frac{2639.2 - 2643.3}{0.6 - 0.5}\right)(0.55 - 0.6) + 2639.2$$

$\therefore y = 2641.3$ kJ/kg

$u_1 = 2641.3$ kJ/kg $\Rightarrow$ u @ 0.55 MPa.

Point ② $\Delta H = 0$.

	H (kJ/kg)	T (°C)		U (kJ/kg)	
x_1	2769.1	150	y_1	2577.1	y_1
x	2853.2	191.4	y	2641.3	$y \equiv u_2$
x_2	2870.2	200	y_2	2654.6	y_2

interpolation for T :

$$y = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_2) + y_2$$

$$y = \frac{200 - 150}{2870.2 - 2769.1}(2853.2 - 2870.2) + 200$$

$$y = 191.4$$

$$\therefore \underline{T = 191.4\ ^\circ C}$$

interpolation for U :

$$y = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_2) + y_2$$

$$= \left(\frac{2654.6 - 2577.1}{2870.2 - 2769.1}\right) \times (2853.2 - 2870.2) + 2654.6$$

$$y = 2641.3$$

$$U_2 = 2641.3\ kJ/kg$$

$\Delta U = U_2 - U_1 = 2641 \cdot 3 - 2641 \cdot 3$

$\Delta U = 0$ kJ/mol.

Q5 Reaction A $\Rightarrow$ stoichiometric

(a)

$$\overset{\nu_1=-1}{1\ CO_{(g)}} + \overset{\nu_2=-2}{2\ H_2\,(g)} \overset{\Delta H_{rxn}}{\rightleftharpoons} \overset{\nu_3=+1}{1\ CH_3OH_{(g)}}$$

	(initial moles) n^i	(final moles) n^f
n_{CO}	1	$1-\xi$
n_{H_2}	2	$2-2\xi$
n_{CH_3OH}	0	$0+\xi$
n total	3	$3-2\xi$

$$n^f = n^i + \nu_i\,\xi$$

ν_i: stoich. coefficient

ξ: Reaction coordinate or extent of rxn.

$$0 < \xi < 1$$

mole fractions (y_i) $\equiv$ $\dfrac{\text{final no. of moles of species}}{\text{Total no. of moles.}}$

y_{CO}	$\dfrac{1-\xi}{3-2\xi}$	← no. of n_{Co}^{f} ← n_{tot}
y_{H_2}	$\dfrac{2-2\xi}{3-2\xi}$	
y_{CH_3OH}	$\dfrac{\xi}{3-2\xi}$	
y_{tot}	1	

(b)

Heat of reaction $\Delta H_{rxn, 298K}$

ν: stoichiometric coefficient.

$\nu_1 = -1$ $\nu_2 = -2$ $\nu_3 = +1$

$$1\ CO_{(g)} + 2\ H_2(g) \xrightleftharpoons{\Delta H_{rxn}} 1\ CH_3OH_{(g)}$$

① ② ③

$\Delta H_{f,298} = -110.53$ kJ/mol

$\Delta H_f = 0$

$2H_2$

$\Delta H_{f,298} = -200.66$ kJ/mol

$C_{(s)} + \frac{1}{2} O_2\ (g)$

(* refer Appendix E from book for ΔH_f values)

$C_{(s)} + \frac{1}{2} O_2\ (g) + 2H_2$

(reference Dortmund data bank)

$$\Delta H_{rxn} = \sum \nu_i \Delta H_f^{i}$$

$$\therefore \Delta H_{rxn} = \nu_3 \Delta H_f^{1} + \nu_1 \Delta H_f^{1} + \nu_2 \Delta H_f^{2}$$

$$= -200.65 + 110.53 - 2 \times (0)$$

$$\therefore \Delta H_{rxn} = -90.12 \text{ kJ/mol}$$

Reaction B → 1 mol of C_2H_5OH ← feed ratio

(a)

$\nu_1 = -1$ $\qquad\qquad$ $\nu_2 = +1$ $\qquad$ $\nu_3 = +1$

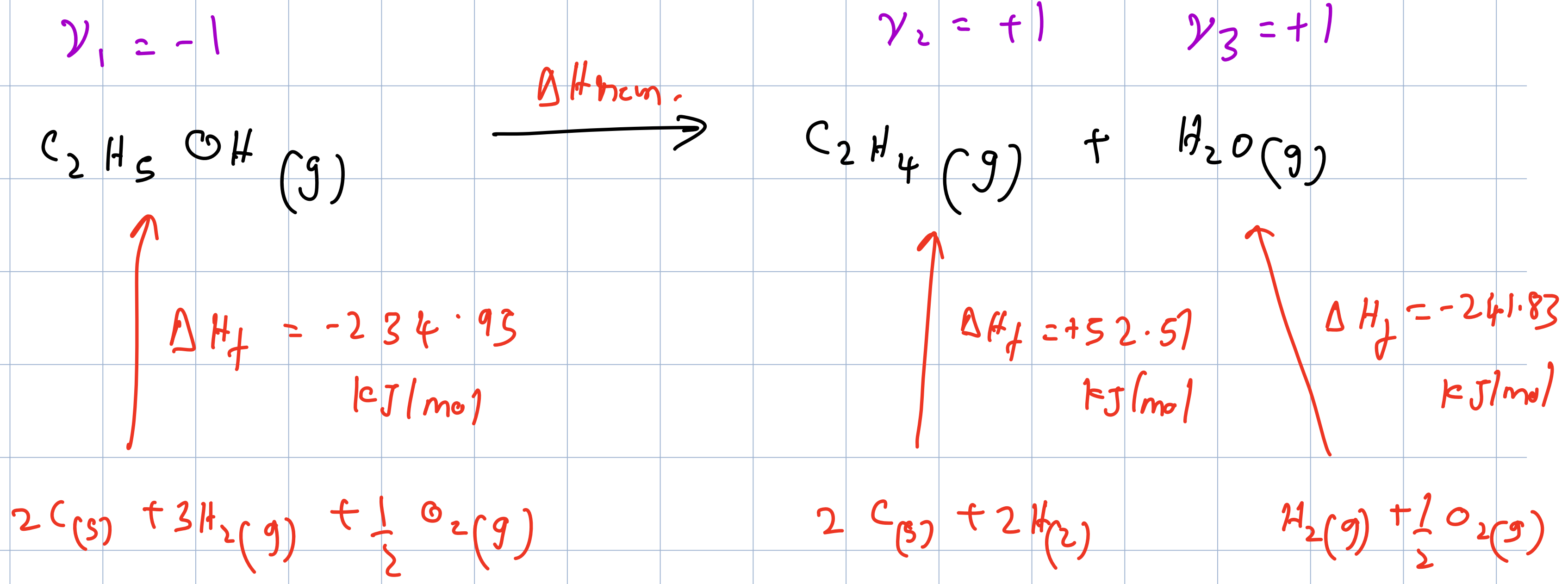
$$C_2H_5OH_{(g)} \longrightarrow C_2H_4(g) + H_2O(g)$$

$$n^f = n^i + \nu_i \xi$$

	n^i	n^f
$n_{C_2H_5OH}$	1	$1 - \xi$
$n_{C_2H_4}$	0	$0 - \xi$
n_{H_2O}	0	$0 + \xi$
n_{total}	1	$1 + \xi$

$0 < \xi < 1$

(b)



$$\Delta H_{rxn} = \sum \nu_i \, \Delta H_f^{\,i}$$

$$\therefore \Delta H_{rxn} = -1(-234.95) + 52.51 + (-241.835)$$

$$\therefore \Delta H_{rxn} = 45.625 \text{ kJ/mol}$$

Reaction C → Stiochiometric feed

$$\overset{\nu_1 = -1}{CO_{2\,(g)}} + \overset{\nu_2 = -4}{4H_{2\,(g)}} \rightleftharpoons \overset{\nu_3 = +1}{CH_{4\,(g)}} + \overset{\nu_4 = +2}{2H_2O_{(g)}}$$

(a)

	n^i	n^f
n_{CO_2}	1	$1 - \xi$
n_{H_2}	4	$4 - 4\xi$
n_{CH_4}	0	$0 + \xi$
n_{H_2O}	0	$0 + 2\xi$
n_{tot}	5	$5 - 2\xi$

$$0 < \xi < 1$$

(b)

$$\Delta H_{rxn} = -\overset{CO_2}{(-393.51)} - \overset{H_2}{0}$$

$$\overset{CH_4}{+(-74.89)} + 2\overset{H_2O}{(-241.8)}$$

$$\Delta H_{rxn} = -165.0536 \text{ kJ/mol.}$$

Reaction D → Feed = 3 mol CO, 4 mol H_2O

$$\overset{\nu_1 = -1}{CO_{(g)}} + \overset{\nu_2 = -1}{H_2O\ (g)} \rightleftharpoons \overset{\nu_3 = +1}{CO_{2(g)}} + \overset{\nu_4 = +1}{H_2\ (g)}$$

(a)

	n^i	n^f
n_{CO}	3	$3 - \xi$
n_{H_2O}	4	$4 - \xi$
n_{CO_2}	0	$0 + \xi$
n_{H_2}	0	$0 + \xi$
n_{tot}	7	7

$$0 < \xi < 3$$

(b)

$$\Delta H_{rxn} = -\overset{CO}{(-110.53)} - \overset{H_2O}{(-241.835)} + \underset{CO_2}{(-393.51)} + \underset{H_2}{0}$$

$$\Delta H_{rxn} = -41.145 \text{ kJ/mol}$$