Thermo

Recitation #5

(03/07/2023)

Stirling's Approximation

$$\ln(N!) \approx N\ln(N) - N$$

works for $N \gg 1$

$N = 200$ ~ 0.5% error

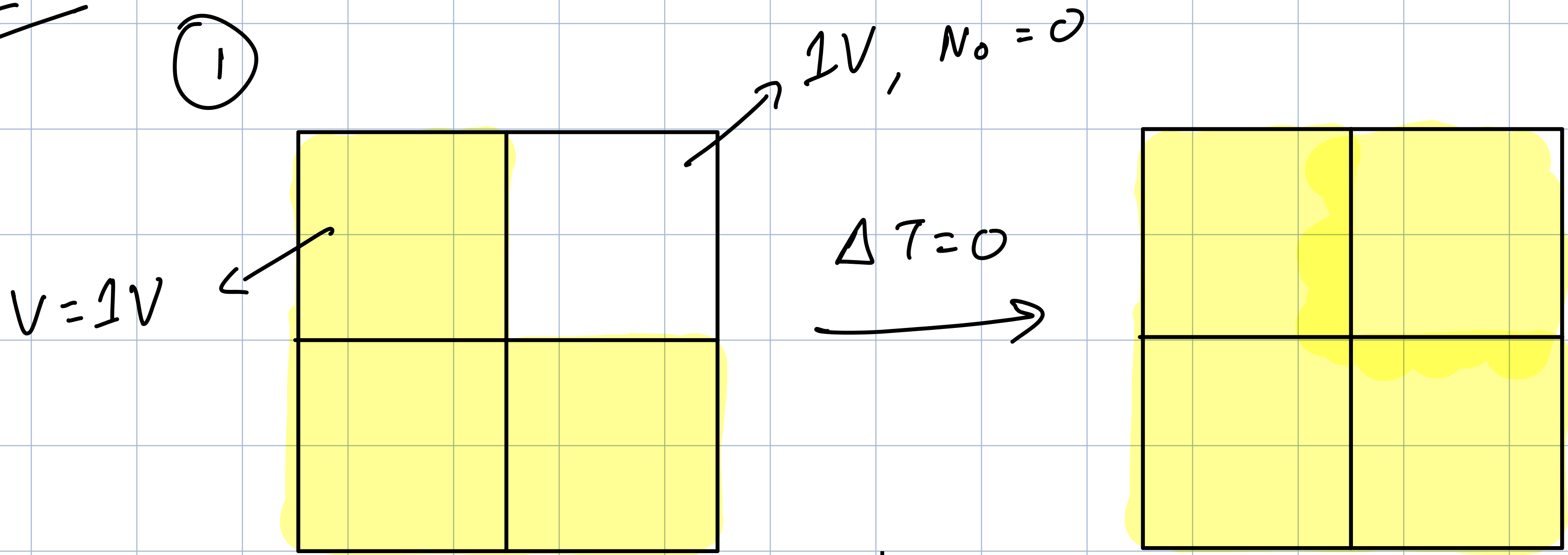
∴ scale of molecules allows this to be a good approximation.

$$S = k_B \ln(P)$$

books also write (Ω or W) instead of P

Q1

①



N Particles total

$V = 3V$ total volume

Total	Per box
$N = N$	$N_B = N/3$
$m = 3$	$N_0 = 0$

N Particles total

$V = 4V$ total volume

Totals	Per box
$N = 4$	$N_B = N/4$
$m = 4$	

$$P_1 = \frac{N!}{\prod m_{ij}!}$$

$$= \frac{N!}{\left(\frac{N}{3}\right)!\left(\frac{N}{3}\right)!\left(\frac{N}{3}\right)!\,\cancel{0!} \rightarrow 1}$$

$$P_2 = \frac{N!}{\left(\left(\frac{N}{4}\right)!\right)^4}$$

we know,

$$S = k_B \ln(P)$$

$$\therefore \Delta S = S_2 - S_1 = k_B \ln(P_2) - k_B \ln(P_1)$$

$$\therefore \Delta S = k_B \ln\left(\frac{P_2}{P_1}\right)$$

$$\therefore \quad \Delta S = k_B \ln \left[\frac{\cancel{N!} / (N/4)!^{4}}{\cancel{N!} / (N/3)!^{3}} \right]$$

$$= k_B \left[3 \ln \left(\frac{N}{3}! \right) - 4 \ln \left(\frac{N}{4}! \right) \right]$$

using Stirling's approximation,

$$\ln \left(\frac{N}{3}! \right) = \frac{N}{3} \ln \left(\frac{N}{3} \right) - \frac{N}{3}$$

$$\ln \left(\frac{N}{4}! \right) = \frac{N}{4} \ln \left(\frac{N}{4} \right) - \frac{N}{4}$$

$$\therefore \quad \Delta S = k_B \left[N \ln\left(\frac{N}{3}\right) - \cancel{N} - \left(N \ln\left(\frac{N}{4}\right) - \cancel{N} \right) \right]$$

$$= k_B \left[N \ln\left(\frac{N}{3}\right) - N \ln\left(\frac{N}{4}\right) \right]$$

$$= N k_B \ln\left(\frac{N/3}{N/4}\right)$$

$$\Delta S = N k_B \ln\left(\frac{4}{3}\right)$$

$$\Delta S = N k_B \ln\left(\frac{V_2}{V_1}\right)$$ $\Rightarrow$ generalizing

for ideal gas & isothermal process

$$V = \frac{nRT}{P}, \quad T_1 = T_2$$

$$\therefore \frac{V_2}{V_1} = \frac{\cancel{nRT_2}/P_2}{\cancel{nRT_1}/P_1} = \frac{P_1}{P_2}$$

$$\therefore \frac{V_2}{V_1} = \frac{P_1}{P_2}$$

and,

$$N k_B = nR$$

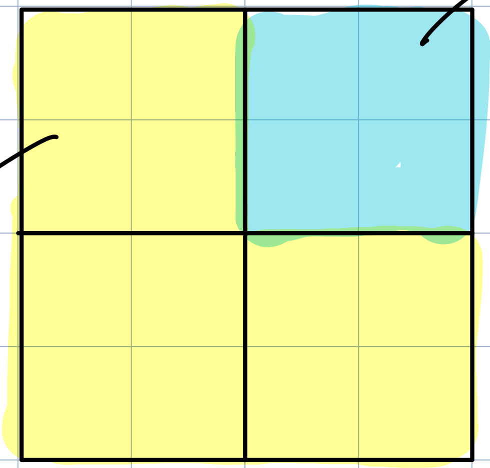
$$\therefore \boxed{(\Delta S)_T = nR \ln\left(\frac{P_1}{P_2}\right) = nR \ln\left(\frac{V_2}{V_1}\right)}$$

$\therefore (\Delta S)_T \uparrow$ if $V \uparrow$ or $P \downarrow$

Q2

O_2, $V_1^{O_2} = 1V$

$n_1^{O_2} = 1\,mol$

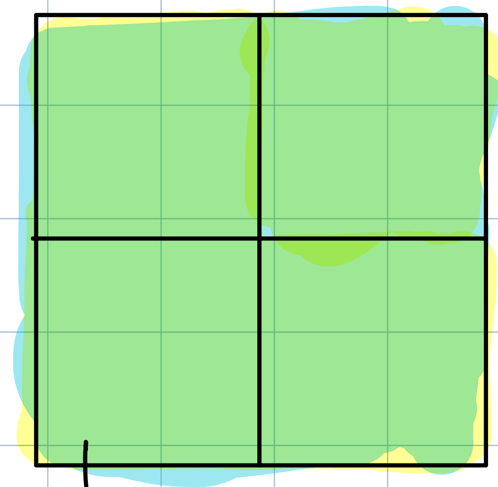


$V_1^{N_2} = 3V$

$n_1^{N_2} = 3\,mol$

$n_{Total} = 4\,mol$

mixing



$V_2^{O_2} = 4V$

$V_2^{N_2} = 4V$

Assuming : Ideal gas & $\Delta T = \Delta P = 0$

$$\Delta S_{total} = \Delta S_{O_2} + \Delta S_{N_2}$$

$$\Delta S_{total} = n_{O_2} R \ln\left(\frac{V_2^{O_2}}{V_1^{O_2}}\right) + n_{N_2} R \ln\left(\frac{V_2^{N_2}}{V_1^{N_2}}\right)$$

$$\therefore \Delta S_{total} = 1\,R\ln\left(\frac{4V}{1V}\right) + 3\,R\ln\left(\frac{4V}{3V}\right)$$

$$= R\left[\ln 4 + 3\ln\frac{4}{3}\right]$$

$$\therefore \Delta S_{total} = 18.7\ J\ K^{-1}$$

General form

$$\Delta S_{total} = \Delta S_{O_2} + \Delta S_{N_2} = \Delta S_{mix}$$

$$\Delta S_{total} = n_{O_2} R\ln\left(\frac{V_2^{O_2}}{V_1^{O_2}}\right) + n_{N_2} R\ln\left(\frac{V_2^{N_2}}{V_1^{N_2}}\right)$$

$$n_{O_2} = x_{O_2}\, n_{total} \qquad V_1^{O_2} = x_{O_2}\, V_{total}$$

$$n_{N_2} = x_{N_2}\, n_{total} \qquad V_1^{N_2} = x_{N_2}\, V_{total}$$

$$V_2^{O_2} = V_2^{N_2} = V_{total}$$

$$\therefore \Delta S_{total} = R\left[x_{O_2}\, n_{total} \ln\left(\frac{V_{\cancel{total}}}{x_{O_2}\, V_{\cancel{total}}}\right) + x_{N_2}\, n_{total} \ln\left(\frac{V_{\cancel{total}}}{x_{N_2}\, V_{\cancel{total}}}\right)\right]$$

$$\therefore \Delta S_{total} = n_{total}\, R\left[x_{O_2} \ln\left(\frac{1}{x_{O_2}}\right) + x_{N_2} \ln\left(\frac{1}{x_{N_2}}\right)\right]$$

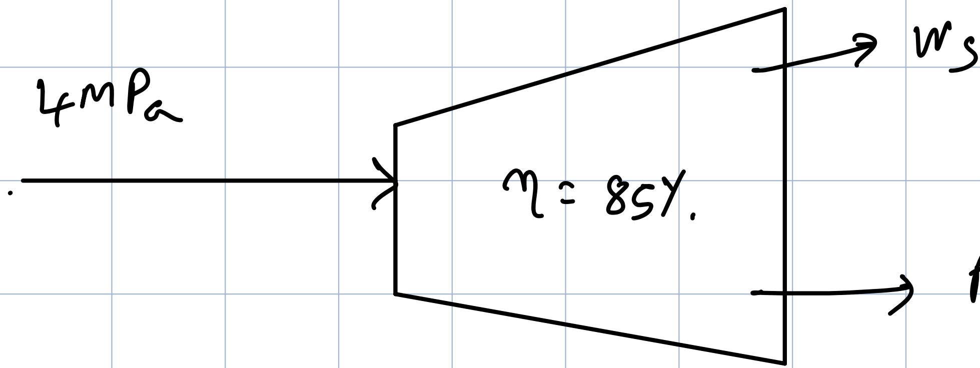
$$\therefore \Delta S_{total} = -n_{total}\, R\left[x_{O_2} \ln\left(x_{O_2}\right) + x_{N_2} \ln\left(x_{N_2}\right)\right]$$

$$\therefore \Delta S_{mix} = -n_{total} R \sum_i x_i \ln(x_i)$$

Q3

Superheated steam.

$P_1 = 4MPa$



$T @ P^{sat} = 48.81\,°C$

$S^v = 8.1488\ kJ/kgK$

$H^v = 2583.86\ kJ/kgK$

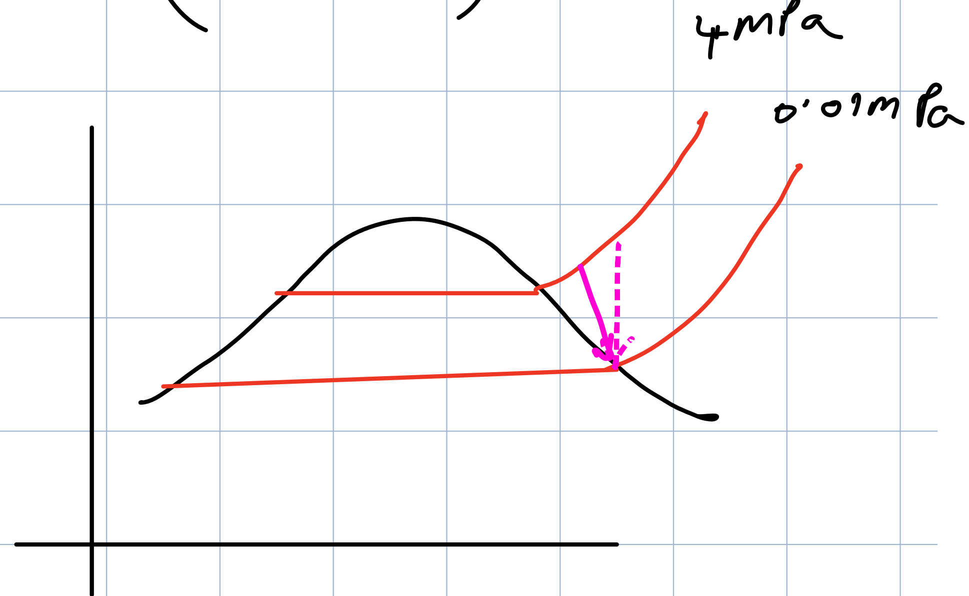
$P_2 = 0.01\ MPa$

$q = 100\%$

($\because$ saturated)

Assume → Adiabatic

steady state.



Energy balance

0 (adiabatic)

$$\Delta H' = \cancel{Q} + W_s'$$

$$\Delta H' = W_s'$$

$$\Delta H = \Delta H' \eta$$

$$\eta = \frac{\underline{W}_s}{\underline{W}_s'}$$

Outlet

Adiabatic & reversible process ≡ isentropic process.

$P_2 = 0.01$ MPa

$T^{sat} @ P_2 = 48.81\ ^\circ C$

$S^v = 8.1488$ kJ/kg·K $= S_2$ ← but in this process $S_2 < S_1$ because irrev. work transfer.

$H^v = 2583.86$ kJ/kg K

$H_2 = \Delta H' \eta + H_1$

@ $P^{sat} = 0.01$ MPa

S^L	0.6492
S^V	8.1488
H^L	191.81
H^V	2583.86

inlet

$P_2 = 4$ MPa

Pick a T with → $S < S_2$ } because work transfer is irrev. i.e efficiency (η) < 100%

let $T_2 = 700°C$

$$q = \frac{S_1 - S^L}{\Delta S^{vap}} = 0.93$$

$$H_2' = H^L + q\,\Delta H^{vap}$$

$$H_2' = 2413.04$$

$$\Delta H' = 2413.64 - 3906.3$$

$$= -1490.66$$

$$\Delta H = \Delta H'\eta = -1267.06$$

$$H_2 = \Delta H + H_1 = -1267.06 + 3906.3$$

$$= 2639.2$$

$$q = \frac{H_2 - H^L}{\Delta H^{vap.}} = \underline{\underline{1.02}}$$

$T = 650$

T	650
H	3790.1
S	7.4988

if $S_1 = S_2$

$$q = \frac{S_1 - S^l}{\Delta S^{vap}} = 0.91$$

$$H_2' = H^l + q\,\Delta H^{vap}$$
$$= 2376.54$$

$$\Delta H' = 2376.54 - 3790.1$$
$$= -1413.56$$

$$\Delta H = \Delta H' \eta = -1201.53$$

$$H_2 = \Delta H + H_1 = -1201.53 + 3790.1$$
$$= 2588.6$$

$$H_2 \approx H^v \Rightarrow q \approx 1.00$$

$\therefore$ $$\boxed{\begin{array}{l} \text{superheat at inlet} = T = 650^\circ C \\ \text{work done by turbine} = W_s' = \Delta H' = -1413.56\ kJ/kg \end{array}}$$