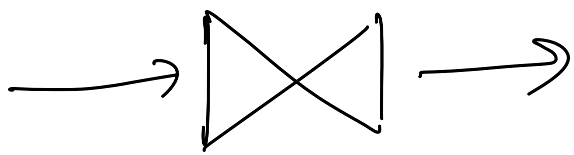
Thermo

Recitation #7

03/21/23

Q 1

1



2

$T_1 = 500K$

$P_2 = 0.5 MPa$, $\dot{n}_2 = 3 mol/min$

$P_1 = 3 MPa$

$\dot{n}_1 = 3 mol/min$

Assumption: ideal gas

Steady-state

Adiabatic

isenthalpic

☆ common assumption:

for Throttle valve → isenthalpic & Adiabatic
($\Delta H = 0$) ($Q = 0$)

for steam turbine → only adiabatic
($Q = 0$)

$\Rightarrow$

Energy balance

$$0 = \dot{n} H^{in} - \dot{n} H^{out} + \cancel{\underline{Q}}^{\,0} + \cancel{\underline{W}}^{\,0}$$

$$0 = \dot{n}\,\Delta H \Rightarrow \text{isenthalpic}$$

$$\Delta H = c_p \Delta T = 0$$

$$\therefore \Delta T = 0$$

$\therefore$ isothermal

$$\therefore T_2 = 500\,K$$

Entropy balance

$$\frac{dS}{dt} = \overbrace{\sum s^{in} \dot{n}_1 - \sum s^{out} \dot{n}_2}^{-\Delta s \dot{n}}$$

$$+ \cancel{\frac{\underline{\dot{Q}}}{T_{sys}}}^{\,0} + \underline{\dot{S}}_{gen}$$

$$\underline{\dot{S}}_{gen} = \dot{n}\,\Delta S$$

$$(\Delta S)_T = -R \ln\left(\frac{P_2}{P_1}\right)$$

$$= -8.314 \ln\left(\frac{0.5}{3}\right)$$

$$(\Delta S)_T = 14.9\ J/mol\,K$$

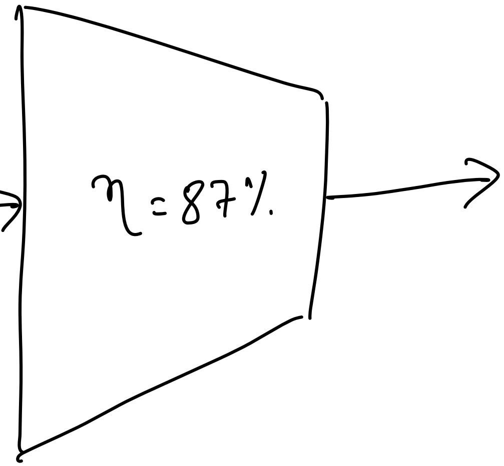
$$\underline{\dot{S}}_{gen} = \dot{n}\,(\Delta S)_T = 14.9 \times 3 = 44.7\ \text{J/min K}$$

since $\dot{S}_{gen} > 0 \Rightarrow$ not reversible

Q2 Methane

$P_1 = 0.1013$ MPa

$T_1 = -240°F$



$P_2 = 0.4$ MPa

Assume: Steady state

Adiabatic

Find: $\underline{W}$ (kJ/mol), $\underline{\dot{W}}_s$ (kJ/s), $\underline{\dot{S}}_{gen}$ (kJ/K hr)

⇒ First assuming ideal compressor i.e $\Delta S = 0$ (isentropic)

$$S_1 = S_2'$$

$H_1 = 840$ kJ/kg (intersection of inlet pressure & 240°F isotherm line)

$S_1 = 2.325$ Btu/lb F $= S_2'$ (value of the isentropic line passing through point 1)

↳(Avg. of 2.35 & 2.45)

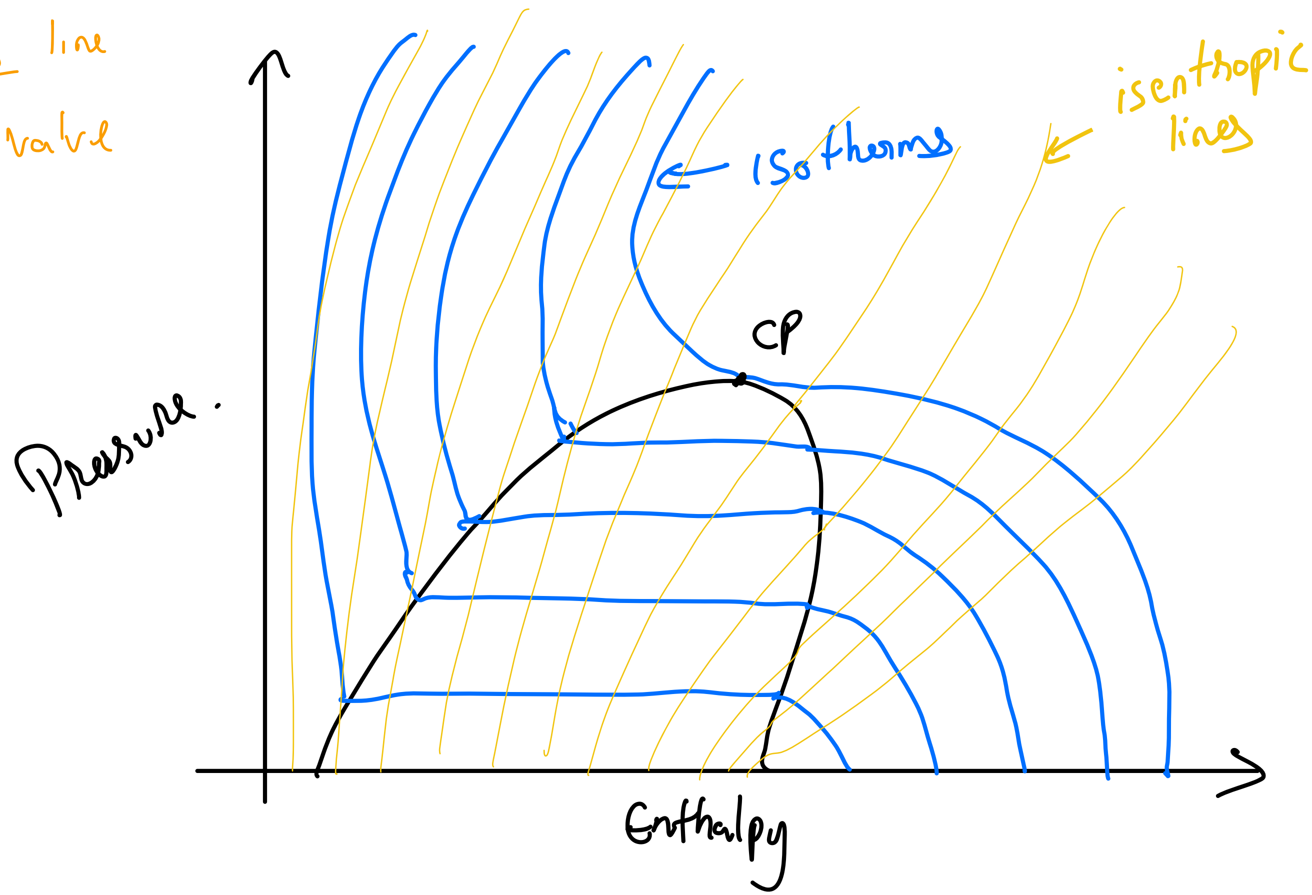
$H_2' = 920$ kJ/kg

(intersection of S_1 & P_2 line give point 2. And y-value of it gives H_2)

E.10: for Methane.

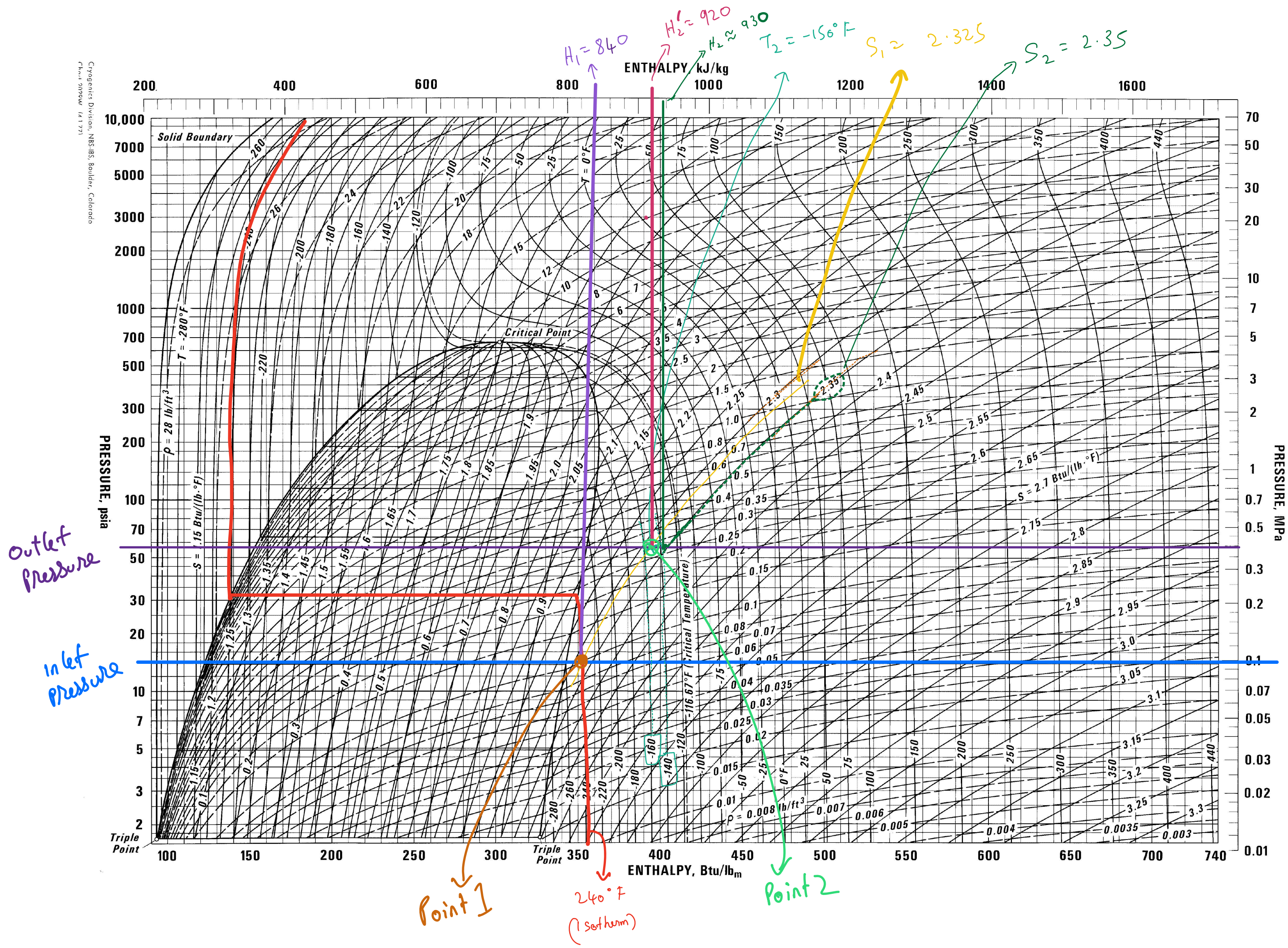
$T_2 = -150°F$

(avg of -140 & -160)



E.10 PRESSURE-ENTHALPY DIAGRAM FOR METHANE

(Source: NIST, Thermophysics Division, Boulder, CO, USA, used with permission.)



(a) $S_1 = S_2'$

$W_s' = \Delta H = H_2' - H_1 = 80 \text{ kJ/kg}$

Actual work :

$W_s' = W_s \cdot \eta$

$\therefore W_s = \dfrac{W_s'}{\eta} = \dfrac{80}{0.87} = 91.95 \text{ kJ/kg}$

$m.W \text{ of } CH_4 = 16.04 \text{ kg/mol}$

$W_s = 1474.88 \text{ kJ/kmol}$

$H_2 = W_s + H_1 = 91.95 + 840 = 931.95 \text{ kJ/kg} \simeq 930$

$S_2 = 2.35 \text{ Btu/lb F}$ (from graph $\rightarrow$ H_2 & P)

(b) $\dot{W}_s = W_s \cdot \dot{n} = 1475.88 \times 17.5 \times \frac{1 \text{ hr}}{60 \times 60 \text{ s}} = 7.17 \text{ kW}$

(c) Entropy Balance

$$\cancel{\frac{dS}{dt}}^{\,0} = \dot{m}\left(s^{in} - s^{out}\right) + \cancel{\frac{Q}{T_{sys}}}^{\,0} + \dot{S}_{gen}$$

$$\dot{S}_{gen} = \dot{m}\left(s_2 - s_1\right)$$

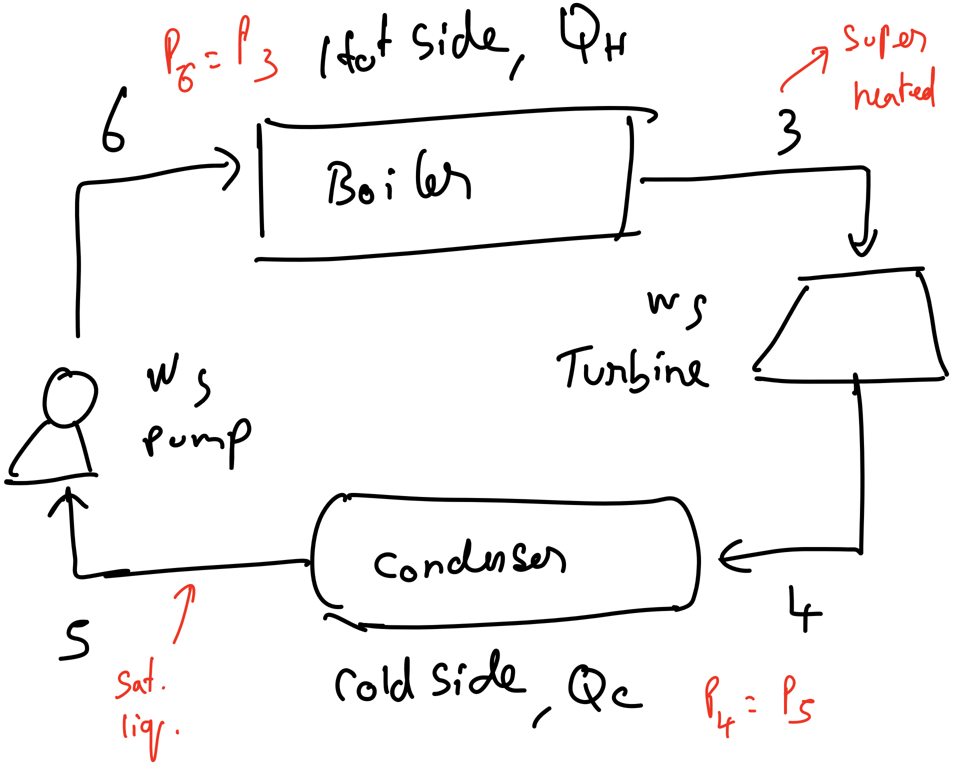
$$= 17.5 \times 16.04 \underbrace{\left(2.35 - 2.325\right)}_{0.025 \text{ Btu/lb F}}$$

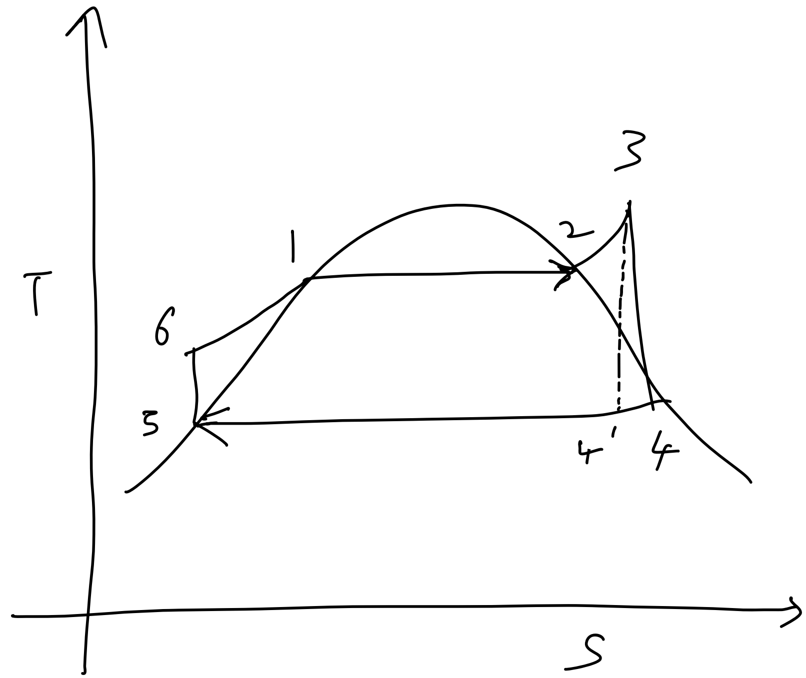
$$\dot{S}_{gen} = 29.38 \text{ kJ/K hr}$$

$1 \text{ kJ/kg K} = 0.23886 \text{ Btu/lb F}$

$0.025 \text{ Btu/lb F} = 0.1047 \text{ kJ/kg K}$

Q3





Given

$T_3 = 600°C$

$P_3 = 10\ MPa$

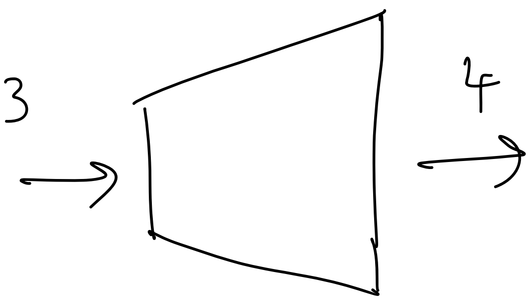
$P_4 = 0.07\ MPa$

$\eta_{turbine} = 0.8$

$\eta_{pump} = 0.75$

Plant Capacity = 80 MW

(a) Find W_s of the turbine



$T_3 = 600°C$

$P_3 = 10\,MPa$

$P_4 = 0.01\,MPa.$

Assume ideal operation

$S_3 = S_4' = 6.9045\ kJ/kg\,K$

@ $P_4 = 0.01\,MPa \rightarrow$ sat. pressure table

$$\left.\begin{aligned} S^L &= 0.6492 \\ S^V &= 8.1488 \end{aligned}\right\}\ kJ/kg\,K$$

$S^L < S_4' < S^V \rightarrow$ quality $< 100\%$.

$$q' = \frac{S_4' - S^L}{S^V - S^L} = \frac{6.9045 - 6.6492}{8.1488 - 0.6492}$$

$$q' = 0.83$$

$$H_4' = H^L + q^L\left(H^V - H^L\right)$$

$$= 191.81 + 0.83\left(2583.86 - 191.81\right)$$

$$H_4' = 2177.21 \text{ kJ/kg}$$

$$W_s' = \Delta H' = H_4' - H_3$$
$$= 2177.21 - 3625.8$$
$$W_s' = -1448.59 \text{ kJ/kg}$$

Actual work :

$$W_s = \Delta H = W_s' \eta_{turbine}$$
$$= -1448 \times 0.8$$
$$W_s = -1158.87 \text{ kJ/kg}$$

work output of the turbine.

$$H_4 = \Delta H + H_3$$
$$= -1158.87 + 3625.8$$
$$H_4 = 2466.93 \text{ kJ/kg}$$

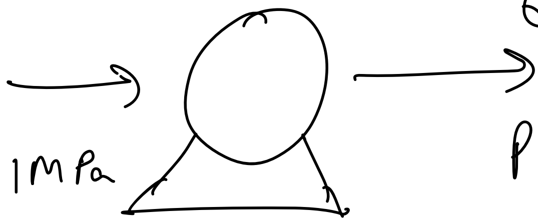
$$q = \frac{H_4 - H^L}{H^V - H^L} = \frac{2466.93 - 191.8)}{2583.86 - 191.8)}$$
$$q = 0.95$$

$$S_4 = S^L + q(S^V - S^L)$$
$$= 0.6492 + 0.95 \times (8.1488 - 0.6492)$$
$$S_4 = 7.774 \text{ kJ/kg.K}$$

(b)

Sat liq.　　　　　　　　　　liq.

5　　　　　　　　　　　　6



$P_5 = P_4 = 0.01\,MPa$　　　　$P_6 = P_3 = 10\,MPa$

∵ P const. across condenser.　　　∵ P const. across boiler

Since sat. liq. at 5

$H_5 = H^L @ P = 0..1\,MPa$

$= 191.81\,kJ/kg$

$V_5 = V^L @ P = 0.01\,MPa$

$= 0.001010\,m^3/kg$

Assume ideal

$$\Delta H'_{pump} = W'_{s,pump} = \Delta PV = V \Delta P$$

$$\therefore \Delta H'_{pump} = 0.001010\,(10 - 0.01) \times \frac{10^6\ Pa}{1\ MPa} \times \frac{1\ kJ}{10^3\ J}$$

$$\frac{m^3}{kg} \qquad MPa$$

$$W_{s,pump} = \frac{W'_{s,pump}}{\eta_{pump}} = \frac{10.09}{0.75} = \boxed{13.45\ kJ/kg = \Delta H_{pump}}$$

$$H_6 = \Delta H_{pump} + H_5 = 13.45 + 191.81$$

$$= 205.26\ kJ/kg$$

(c) Flowrate

$$\dot{m} = \frac{\text{Plant capacity}}{\text{net work}} = \frac{\overset{\text{output}}{-80000}\ kJ/s}{(\underset{\text{output}}{-1158.87} + \underset{\text{input}}{13.45})\ \frac{kJ}{kg}}$$

$$\dot{m} = 69.84 \text{ kg/s}$$

(d) $Q_{boiler} = \Delta H_{boiler} = H_3 - H_6 = 3625.8 - 205.26$

$= 3420.54 \text{ kJ/kg}$

$\dot{Q}_{boiler} = \dot{m}\, Q_{boiler} = 69.84 \times 3420.54$

$= 238890.3 \text{ kJ/s}$

$\dot{Q}_{boiler} = 0.24 \text{ GW (gigawatt)}$

(e) Thermal efficiency

$$\eta_a = -\frac{\text{plant capacity}}{\dot{Q}_{boiler}} = \frac{-(-80000)}{238890.3} = 0.33$$

$= 33\%$

(Q4)

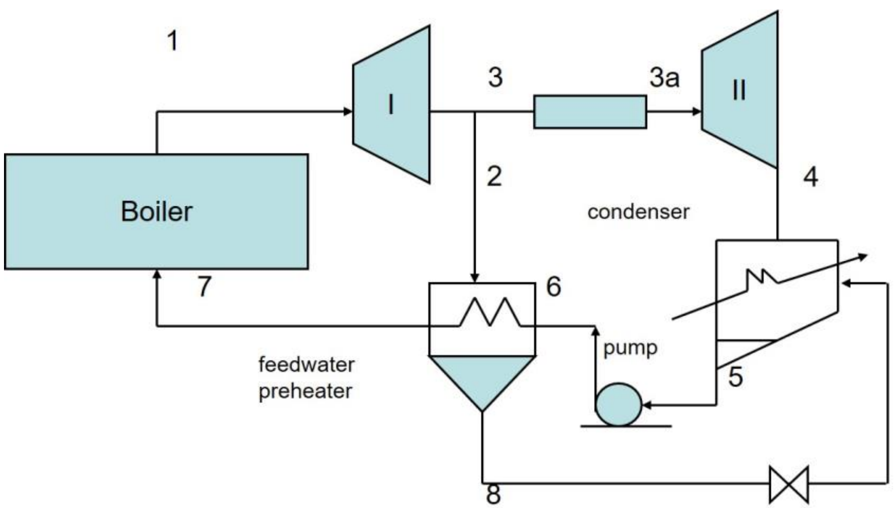
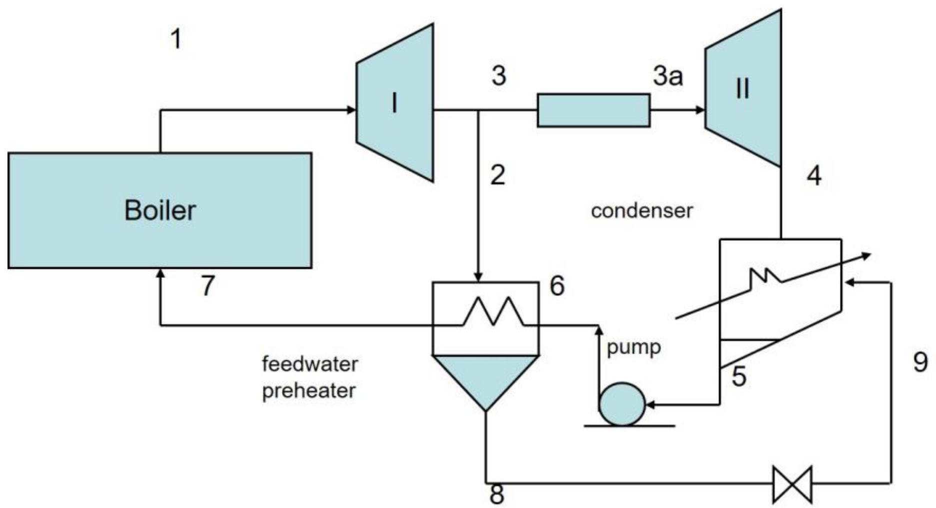


Fig. 5.7 Modified Rankine Cycle with reheat

5.4 A modified Rankine Cycle uses reheat and one closed feedwater preheater. The schematic is a modification of Fig. 5.7 on page 206 obtained by adding a reheater between the T-joint and turbine II. Letting steam 3 denote the inlet to the reheater, and steam 3a denote the inlet to the turbine, the conditions are given below. The plant capacity is to be 80 MW. Other constraints are as follows: The efficiency of each turbine stage is 85%, the pump efficiency is 80%; the feedwater leaving the closed preheater is 5°C below the temperature of the condensate draining from the bottom of the closed preheater. For options below, calculate:

a) the flowrate of stream 1;
b) the thermal efficiency of the plant;
c) the size of the feedwater pump (kW).

T_1 = 500°C, P_1 = 4 MPa, P_2 = 0.8 MPa,
T_{3a} = 500°C, P_4 = 0.01 MPa



Step 1. Make table, add all the given values and find which streams are at the same pressure.

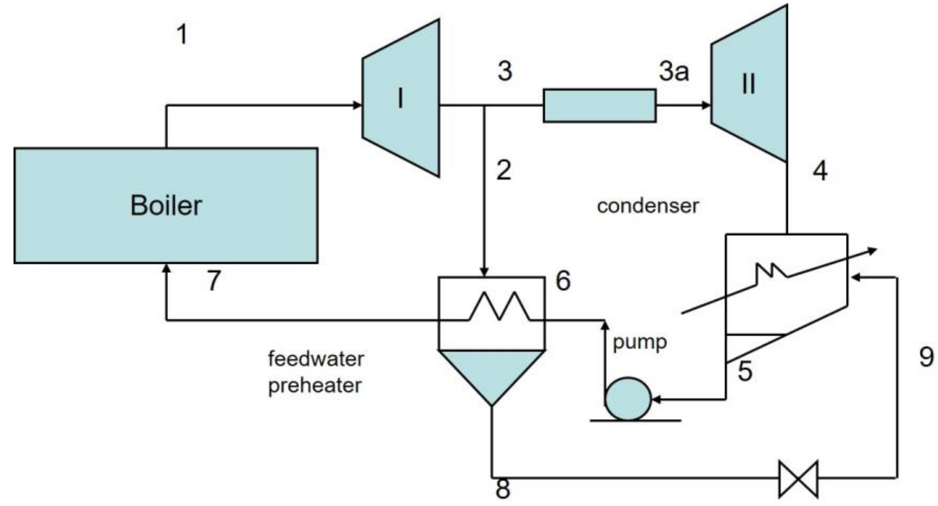
T_1 = 500°C, P_1 = 4 MPa, P_2 = 0.8 MPa,
T_{3a} = 500°C, P_4 = 0.01 MPa

$P_1 = P_6 = P_7$ boiler

$P_2 = P_3 = P_8$ Feedwater preheater

$P_4 = P_5 = P_9$ condenser

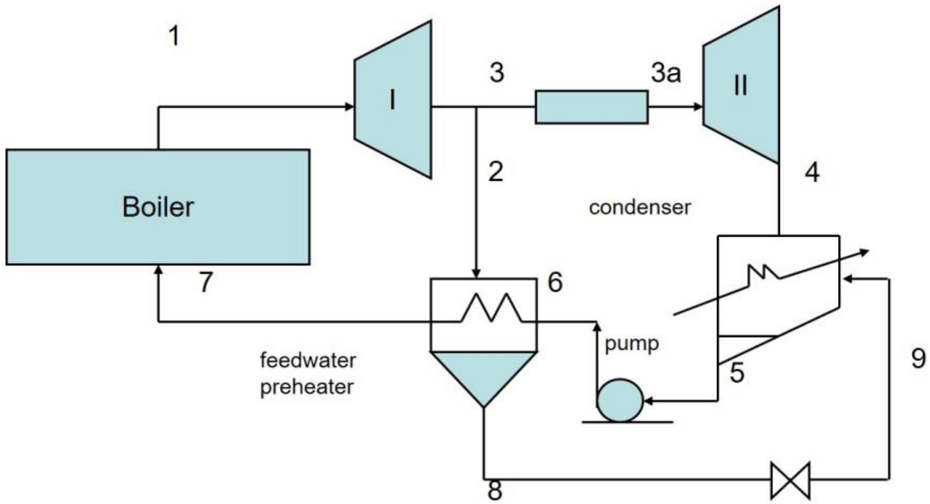
Stream	T [°C]	P [Mpa]	H [kJ/kg]	S [kJ/kgK]
1	500	4		
2/3		0.8		
3a	500	0.8		
4		0.01		
5		0.01		
6		4		
7	T8-5	4		
8	T8	0.8		



Step 2. Determine state of fluids at as many points as you can. For those that are saturated liquid (stream 5&8) you can get H^L and S^L values form the saturation pressure table.

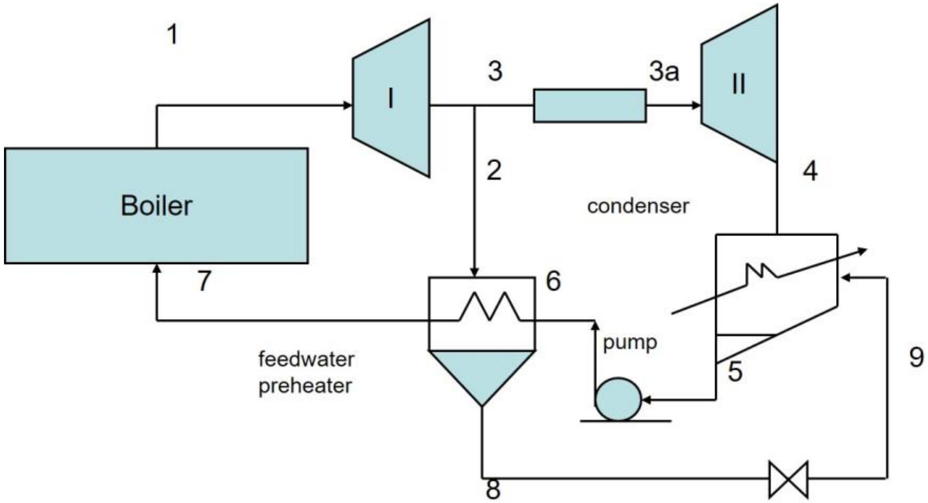
	Stream	T [C]	P [Mpa]	H [kJ/kg]	S [kJ/kgK]
superheated	1	500	4		
	2/3		0.8		
superheated	3a	500	0.8		
	4		0.01		
saturated liquid	5		0.01	191.81	0.6492
	6		4		
compressed liquid*	7	165.41	4		
saturated liquid	8	170.41	0.8	720.86	2.0457

* You can calculate T_7 because you have found T_8 from saturation pressure table.



Step 3. Get the H and S values for streams 1 & 3a.

Stream	T [C]	P [Mpa]	H [kJ/kg]	S [kJ/kgK]
1	500	4	3446.0	7.0922
2/3		0.8		
3a	500	0.8	3481.3	7.8692
4		0.01		
5		0.01	191.81	0.6492
6		4		
7	165.41	4		
8	170.41	0.8	720.86	2.0457



Step 4. Get the H and S values for
Part 1: Reversible outlet (Stream 2'/3') $\Delta S = 0$
Part 2: Irreversible outlet (Stream 2/3)

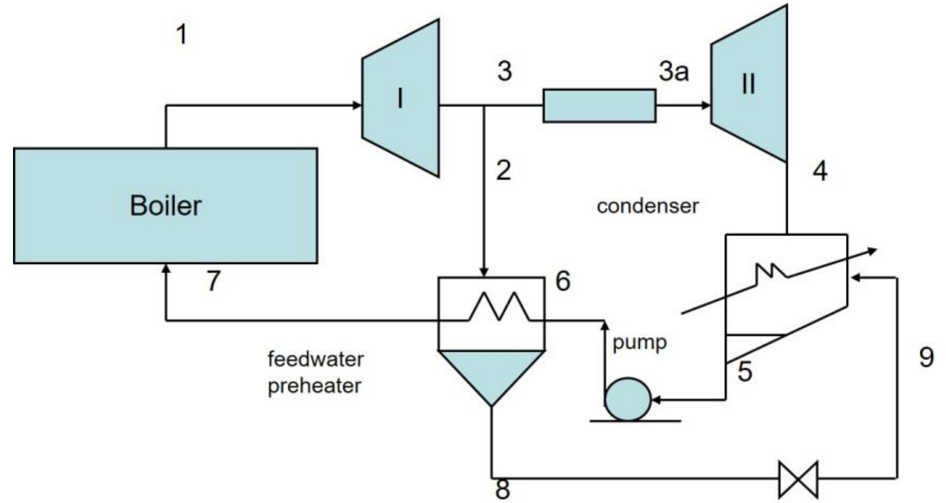
Turbine Efficiency for stage I of 0.85

Stream	T [C]	P [Mpa]	H [kJ/kg]	S [kJ/kgK]
1	500	4	3446.0	7.0922
2'/3'		0.8	2978.83	7.0922
2/3		0.8	3048.91	7.2201*
3a	500	0.8	3481.3	7.8692
4		0.01		
5		0.01	191.81	0.6492
6		4		
7	165.41	4		
8	170.41	0.8	720.86	2.0457

$S_{2/3}^{V} = 6.6616$ kJ/kgK
$S_{2/3}^{V} < S_1$ superheated
Interpolate to get $S_{2'/3'}$ & $H_{2'/3'}$
At 0.8 MPa:
$S'_{250} = 7.0401$ kJ/kgK
$S'_{300} = 7.2345$ kJ/kgK
$H'_{250} = 2950.4$ kJ/kg
$H'_{300} = 3056.9$ kJ/kg
$H_{2'/3'} = 2978.83$ kJ/kg

$\Delta H' = -467.17$ kJ/kg $= W_s'$
$W_s = \eta * W_s' = -397.09$ kJ/kg
$W_s = \Delta H = H_{2/3} - H_1$
$H_{2/3} = \Delta H + H_1 = 3048.91$ kJ/kg

* Use interpolation again.



Step 5. Get the H and S values for
Part 1: Reversible outlet (Stream 4') $\Delta S = 0$
Part 2: Irreversible outlet (Stream 4)

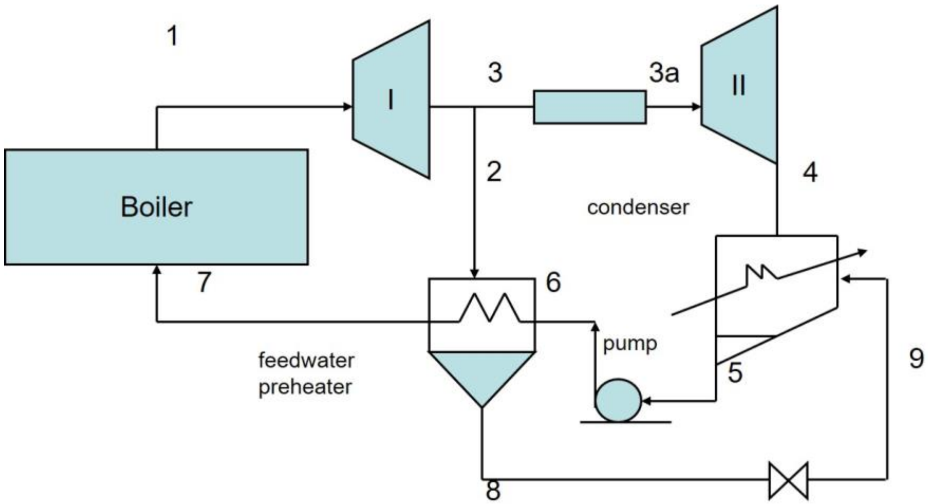
Turbine Efficiency for stage II of 0.85

Stream	T [C]	P [Mpa]	H [kJ/kg]	S [kJ/kgK]
1	500	4	3446.0	7.0922
2'/3'		0.8	2978.83	7.0922
2/3		0.8	3048.91	7.2201
3a	500	0.8	3481.3	7.8692
4'		0.01	2488.18	7.8692
4		0.01	2637.15	8.3040*
5		0.01	191.81	0.6492
6		4		
7	165.41	4		
8	170.41	0.8	720.86	2.0457

* Use interpolation again.

$S_4^V = 8.1488$ kJ/kgK
$S_4^V > S_3$ saturated
Calculate q to get $S_{4'}$ & $H_{4'}$
At 0.01 MPa:
$S_4^L = 0.6492$ kJ/kgK
$S_4^V = 8.1488$ kJ/kgK
q = 0.96
$H_4^L = 191.81$ kJ/kg
$H_4^V = 2583.86$ kJ/kg
$H_{4'} = 2488.18$ kJ/kg

$\Delta H' = -993.12$ kJ/kg $= W_s'$
$W_s = \eta * W_s' = -844.15$ kJ/kg
$W_s = \Delta H = H_4 - H_{3a}$
$H_4 = \Delta H + H_{3a} = 2637.15$ kJ/kg
$H_4^V < H_4$ superheated



Step 6. Calculate H_6 and H_7 using $H_{6'} = H_5^L + V_5^L(\Delta P)$ and $H_7 = H_6^L + V_6^L(\Delta P)$

For $H_{6'}$: $V_5^L = 0.001010\ m^3/kg$
$H_5^L = 191.81$ kJ/kg
$\Delta P = 3.99$ MPa

For H_7: $V_6^L = 0.001108\ m^3/kg$
$H_6^L = 697.24$ kJ/kg
$\Delta P = 4\text{-}0.70090$ MPa

(Note, P_{in} is P^{sat} at 165 °C, V^L is at P^{sat})

Stream	T [C]	P [Mpa]	H [kJ/kg]	S [kJ/kgK]
1	500	4	3446.0	7.0922
2/3		0.8	3048.91	7.2201
3a	500	0.8	3481.3	7.8692
4		0.01	2637.15	8.3040
5		0.01	191.81	0.6492
6’		4	195.84	
6		4	196.85	
7	165.41	4	700.90	
8	170.41	0.8	720.86	2.0457

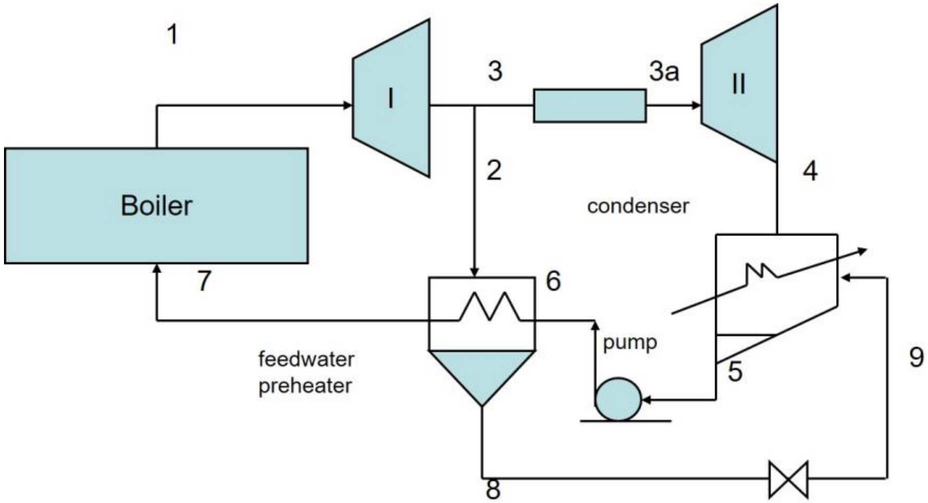
Pump Efficiency = 0.8

$\Delta H' = 4.03$ kJ/kg $= W_s'$
$W_s = W_s'/\eta = 5.04$ kJ/kg
$W_s = \Delta H = H_6 - H_5$
$H_6 = \Delta H + H_5 = 196.85$ kJ/kg

Stream	T [C]	P [Mpa]	H [kJ/kg]	S [kJ/kgK]
1	500	4	3446.0	7.0922
2/3		0.8	3048.91	7.2201
3a	500	0.8	3481.3	7.8692
4		0.01	2637.15	8.3040
5		0.01	191.81	0.6492
6		4	196.85	
7	165.41	4	700.9	
8	170.41	0.8	720.86	2.0457

Step 7. Calculate

$\frac{\dot{m}_2}{\dot{m}_1}, \frac{\dot{m}_{3a}}{\dot{m}_1}$, QB, Qc, and QRH



Use the feedwater preheater EB: $\dot{m}_1(H_7 - H_6) = -\dot{m}_2(H_8 - H_2)$

$$\frac{\dot{m}_2}{\dot{m}_1} = -\frac{(H_7 - H_6)}{(H_8 - H_2)} = -\frac{504.05}{-2328.05} = 0.2165 \qquad \frac{\dot{m}_{3a}}{\dot{m}_1} = \left(1 - \frac{\dot{m}_2}{\dot{m}_1}\right) = 1 - 0.217 = 0.7835$$

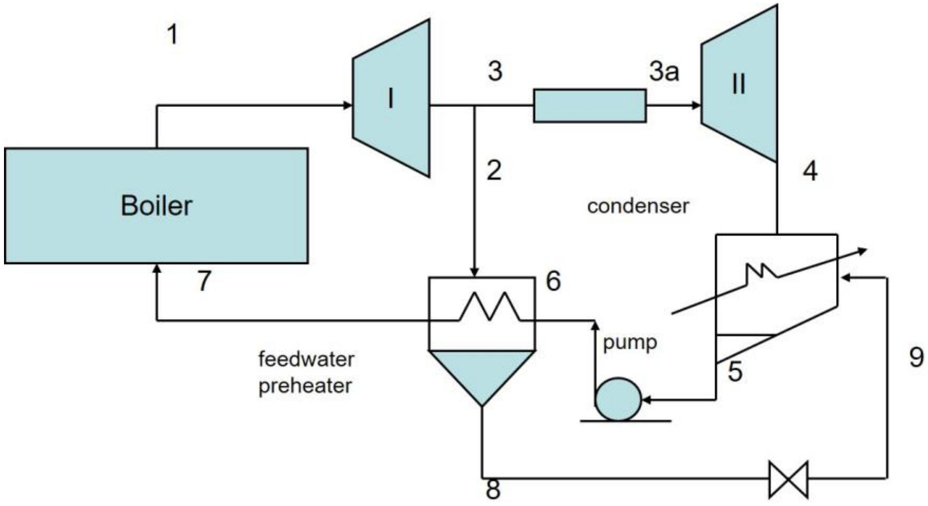
Boiler: $Q_B = (H_1 - H_7) = 2745.1$ kJ/kg stream 1

Condenser: $Q_C = (H_5 - H_4)\frac{\dot{m}_{3a}}{\dot{m}_1} = -2445.34\ kJ/kg * 0.7835$ = -1915.9 kJ/kg stream 1

Reheater: $Q_{RH} = (H_{3a} - H_{2/3})\frac{\dot{m}_{3a}}{\dot{m}_1} = 432.39\ kJ/kg * 0.7835$ = 338.78 kJ/kg stream 1

Stream	T [C]	P [Mpa]	H [kJ/kg]	S [kJ/kgK]
1	500	4	3446.0	7.0922
2/3		0.8	3048.91	7.2201
3a	500	0.8	3481.3	7.8692
4		0.01	2637.15	8.3040
5		0.01	191.81	0.6492
6		4	196.85	
7	165.41	4	700.9	
8	170.41	0.8	720.86	2.0457

Step 8. Calculate $\dot{m}_1$, Power = 80 MW = 80,000 kJ/s



$$\dot{W}_{\text{s,net}} = \dot{m}_1\left(W_{s,turbine\ 1}\right) + \dot{m}_{3a}\left(W_{s,turbine\ 2}\right) + \dot{m}_1\left(W_{s,pump}\right)$$

$$\frac{\dot{W}_{\text{s,net}}}{\dot{m}_1} = \left(W_{s,1}\right) + \frac{\dot{m}_{3a}}{\dot{m}_1}\left(W_{s,2}\right) + \left(W_{s,pump}\right) = \left(W_{s,1}\right) + \frac{\dot{m}_1 - \dot{m}_2}{\dot{m}_1}\left(W_{s,2}\right) + \left(W_{s,pump}\right)$$

$$= \left(W_{s,1}\right) + \left(1 - \frac{\dot{m}_2}{\dot{m}_1}\right)\left(W_{s,2}\right) + \left(W_{s,pump}\right)$$

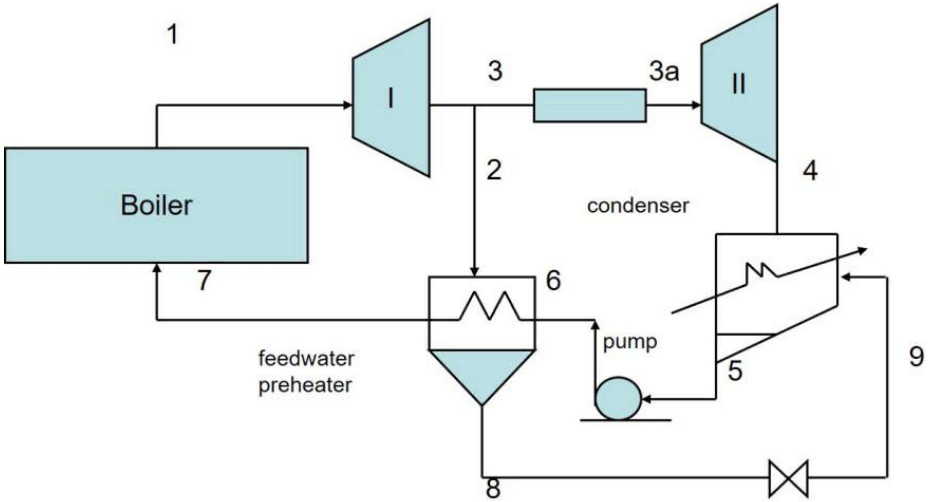
$$= \left(H_2 - H_1\right) + \left(1 - \frac{\dot{m}_2}{\dot{m}_1}\right)\left(H_4 - H_{3a}\right) + \left(H_6 - H_5\right)$$

$$= (-397.09) + (0.7835)(-844.15) + (5.04) = -\frac{1053.44\ kJ}{kg\ stream\ 1}$$

$$\dot{m}_1 = Power/\left(\frac{\dot{W}_{\text{s,net}}}{\dot{m}_1}\right) = \frac{80000\ kJ/s}{\frac{1053.44 kJ}{kg\ stream\ 1}} = \boxed{75.94\ kg/s}$$

Stream	T [C]	P [Mpa]	H [kJ/kg]	S [kJ/kgK]
1	500	4	3446.0	7.0922
2/3		0.8	3048.91	7.2201
3a	500	0.8	3481.3	7.8692
4		0.01	2637.15	8.3040
5		0.01	191.81	0.6492
6		4	196.85	
7	165.41	4	700.9	
8	170.41	0.8	720.86	2.0457

Step 9. Calculate thermal efficiency and size of feedwater pump (kW)



$$Q_{H,total} = (Q_B + Q_{RH})\dot{m}_1 = (2745.1 + 338.78) \text{ kJ/kg stream 1} * 75.94 \text{ kg/s}$$

$$Q_{H,total} = 234189.85 \text{ kJ/s}$$

Thermal Efficiency $\eta = \frac{W_{s,net}}{Q_{H,total}} = \frac{79998.23 kJ/s}{234189.85 kJ/s} = \boxed{0.34}$

$$W_{s,pump} = (H_6 - H_5)\dot{m}_1 = 5.04 \text{ kJ/kg stream 1} * 75.94 \text{ kg/s} = \boxed{382.74 \text{ kW}}$$